Quenching along a gapless line: A different exponent for defect density

Uma Divakaran,[1, *] Amit Dutta,[1, †] and Diptiman Sen[2, ‡]

[1]*Department of Physics, Indian Institute of Technology Kanpur 208 016, India*
[2]*Center for High Energy Physics, Indian Institute of Science, Bangalore 560 012, India*
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We use a new quenching scheme to study the dynamics of a one-dimensional anisotropic XY spin-1/2 chain in the presence of a transverse field which alternates between the values $h + \delta$ and $h - \delta$ from site to site. In this quenching scheme, the parameter denoting the anisotropy of interaction (γ) is linearly quenched from $-\infty$ to $+\infty$ as $\gamma = t/\tau$, keeping the total strength of interaction J fixed. The system traverses through a gapless phase when γ is quenched along the critical surface $h^2 = \delta^2 + J^2$ in the parameter space spanned by h, δ and γ. By mapping to an equivalent two-level Landau-Zener problem, we show that the defect density in the final state scales as $1/\tau^{1/3}$, a behavior that has not been observed in previous studies of quenching through a gapless phase. We also generalize the model incorporating additional alternations in the anisotropy or in the strength of the interaction, and derive an identical result under a similar quenching. Based on the above results, we propose a general scaling of the defect density with the quenching rate τ for quenching along a gapless critical line.

I. INTRODUCTION

The dynamics of a quantum system swept across a quantum critical point at a uniform rate has been studied extensively in recent years. Since a quantum phase transition[1,2] is necessarily accompanied by a diverging correlation length as well as a diverging relaxation time, the dynamics of the system cannot be adiabatic for the entire period of the evolution however slow the variation in the parameter may be. (The relaxation time of a quantum system is given by the inverse of the energy gap which goes to zero at the quantum critical point). Assuming that the system was initially prepared in its ground state, the non-adiabaticity near a quantum critical point prevents the system from following its instantaneous ground state resulting in the production of defects in the final state.

The Kibble-Zurek (KZ) argument[3] asserts that the non-adiabatic effect becomes prominent only close to the critical point when the rate of change of the Hamiltonian is of the order of the relaxation time of the underlying quantum system. When a parameter of the quantum Hamiltonian is varied as t/τ, where τ is the characteristic time scale of the quenching, the above argument predicts a density of of defects in the final state that scales as $1/\tau^{d\nu/(z\nu+1)}$ in the limit of $\tau \to \infty$. Here ν and z denote the correlation length and dynamical exponents, respectively, characterizing the associated quantum phase transition of the d-dimensional quantum system. The KZ prediction has been verified for various exactly solvable spin models when quenched across a critical[4,5] or a multicritical[6] point at a uniform linear rate. The above studies have been generalized to explore the defect production in a non-linear quench across a quantum critical point where a parameter in the Hamiltonian is varied as $|t/\tau|^\alpha$ with $\alpha > 0$[7]. Recent experimental studies on the dynamics of quantum systems[8], espe-

cially quantum magnets[9], ultracold atoms trapped in optical lattices[10] and spin-one Bose-Einstein condensates[11], have paved the way for a plethora of related theoretical studies[4,5,6,7,12,13,14,15,16,17].

Another interesting scenario emerges when a low-dimensional quantum system is quenched *through* a gapless phase or an extended quantum critical region[18,19,20]. It has been established that when a d-dimensional system is quenched along a $(d - m)$-dimensional critical surface, the scaling of the defect density with τ is modified to a generalized KZ form given by $1/\tau^{m\nu/(z\nu+1)}$[19].

In the present work, we explore the dynamics of a one-dimensional anisotropic XY spin-1/2 chain in the presence of a transverse field which alternates between $h + \delta$ and $h - \delta$ from site to site. We employ a new quenching scheme in which the parameter determining the anisotropy of interaction is quenched as t/τ, keeping the strength of interaction fixed, in such a way that the system is driven along a gapless line on a critical surface in the parameter space. We show that the density of defect scales as $1/\tau^{1/3}$, a result that cannot be explained by the previous studies on the quenching through a gapless phase. We also propose a general scaling relation for such a quenching dynamics along a gapless line.

The paper is organized as follows. Our model, the quenching scheme and the results obtained for the generation of defects are presented in Sec. II. At the end of that section, we propose a general scaling relation for the defect density when a system is quenched along a gapless line. We end with some concluding remarks in Sec. III.

II. THE QUENCHING DYNAMICS AND THE RESULT

The Hamiltonian of the spin-1/2 anisotropic XY model with an alternating transverse field is given by[21,22]

$$H = -\frac{1}{2}\Big[\sum_j (J_x + J_y)(\sigma_j^x \sigma_{j+1}^x + \sigma_j^y \sigma_{j+1}^y) +$$

$$+ (J_x - J_y)(\sigma_j^x \sigma_{j+1}^x - \sigma_j^y \sigma_{j+1}^y) + (h - (-1)^j \delta)\sigma_j^z)\Big], (1)$$

where σ's denote the Pauli spin operators satisfying the standard commutation relations. The strength of the transverse field coupled to the operator σ^z alternates between $h + \delta$ and $h - \delta$ on the odd and even sites respectively. We have chosen all the interactions and the fields to be non-random. Henceforth, we shall refer to $J_x + J_y = J$ as the strength and $\gamma = J_x - J_y$ as the anisotropy of the nearest-neighbor interaction.

To map the spin operators to spinless fermion operators using the Jordan-Wigner transformation[23,24], we note that the presence of two underlying sub-lattices necessitates the introduction of a pair of fermion operators a and b[21,22] for even and odd sites as defined below

$$\sigma_{2j}^+ = b_{2j}^\dagger \exp[i\pi \sum_{l=1}^{j-1} b_{2l}^\dagger b_{2l} + i\pi \sum_{l=1}^{j} a_{2l-1}^\dagger a_{2l-1}],$$

$$\sigma_{2j+1}^+ = a_{2j+1}^\dagger \exp[i\pi \sum_{l=1}^{j} b_{2l}^\dagger b_{2l} + i\pi \sum_{l=0}^{j-1} a_{2l+1}^\dagger a_{2l+1}]. (2)$$

Using a restricted zone scheme (where the wave vector k ranges from $-\pi/2$ to $\pi/2$) in the Fourier space, the Hamiltonian can be written as

$$H = \sum_k H_k = \sum_k \hat{A}_k^\dagger \hat{H}_k \hat{A}_k,$$

where $\hat{A}_k$ is the column $(a_k^\dagger, a_{-k}, b_k^\dagger, b_{-k})$. The 4×4 Hermitian matrix H_k is given by

$$\begin{bmatrix} h + J\cos k & i\gamma \sin k & 0 & -\delta \\ -i\gamma \sin k & -h - J\cos k & \delta & 0 \\ 0 & \delta & J\cos k - h & i\gamma \sin k \\ -\delta & 0 & -i\gamma \sin k & -J\cos k + h \end{bmatrix}. (3)$$

The excitation spectrum of the Hamiltonian H is now obtained by diagonalizing the reduced Hamiltonian matrix H_k and is given by

$$\Lambda_k^\pm = [h^2 + \delta^2 + J^2\cos^2 k + \gamma^2\sin^2 k$$

$$\pm 2\sqrt{h^2\delta^2 + h^2 J^2 \cos^2 k + \delta^2 \gamma^2 \sin^2 k}]^{1/2}. (4)$$

Denoting the four eigenvalues by $\pm\Lambda_k^\pm$, we can write the spectrum of the Hamiltonian in the form

$$H = \sum_{-\pi/2 < k < \pi/2} \sum_{\nu = +, -} \Lambda_k^\nu (\eta_{k,\nu}^\dagger \eta_{k,\nu} - \frac{1}{2}), (5)$$
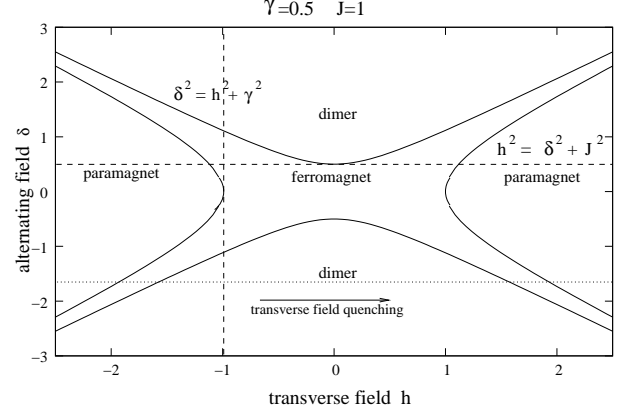

FIG. 1: Phase diagram of the XY chain in an alternating transverse field. We have chosen $J = 1$ and the critical lines are drawn in the $\gamma = 0.5$ plane. Two special points ($h = \pm J$, $\delta = 0$) and ($h = 0, \delta = \pm\gamma$) are shown on the phase boundaries. The spin chain undergoes a quantum phase transition with $\nu = 2$ and $z = 1$ when these points are approached along the dashed line. On the other hand, the dotted line shows the direction of the quenching of the transverse field. We quench the system along a gapless line parallel to the γ-axis (perpendicular to the plane of the paper) passing through the phase boundary $h^2 = \delta^2 + J^2$.

where $\eta_{k,\nu}^\dagger$ is the quasiparticle creation operator corresponding to the mode (k, ν). In the ground state the levels $-\Lambda_k^+$ and $-\Lambda_k^-$ are filled. At the quantum critical point, Λ_k^- vanishes at an ordering wave vector, and the critical exponents are obtained by studying the behavior of Λ_k^- in the vicinity of the critical point.

The minimum energy gap in the excitation spectrum occurs at $k = 0$ and $k = \pi/2$. The corresponding phase boundaries given by $h^2 = \delta^2 + J^2$ and $\delta^2 = h^2 + \gamma^2$ signal quantum phase transitions from a paramagnetic to a ferromagnetic phase and a dimer to ferromagnetic phase respectively (see Fig. 1). Let us define a new set of Pauli matrices τ as

$$\tau_i^x = (-1)^i \sigma_i^x, \quad \tau_i^y = \sigma_i^y \quad \text{and} \quad \tau_i^z = (-1)^i \sigma_i^z,$$

so that the commutation relations of the Pauli matrices are preserved. It is interesting to note that under this unitary transformation, we arrive at a set of duality relations given by $h \to -\delta$, $\delta \to -h$, $J \to -\gamma$ and $\gamma \to -J$; this signifies that the ferro-para transition and the ferro-dimer transition at $h^2 = \delta^2 + J^2$ and $\delta^2 = h^2 + \gamma^2$, respectively, are essentially identical, both belonging to the quantum Ising universality class[25] with $\nu = z = 1$. The phase boundary given by $h^2 = \delta^2 + J^2$ with γ arbitrary and J held fixed, defines a critical surface in the parameter space spanned by h, δ and γ. Similarly, the phase boundary $\delta^2 = h^2 + \gamma^2$ with arbitrary J once again defines another critical surface when γ is held fixed. For $\gamma = 0$, a gapless phase exists with an ordering wave vector $\cos k = \sqrt{h^2 - \delta^2}/J$ for $\delta^2 < h^2 < \delta^2 + J^2$. The system undergoes a quantum phase transition from a gapless

phase to a gapped phase when $|h|$ is increased beyond the critical value given by $h_c = \sqrt{\delta^2 + J^2}$.

The special case with $\delta = 0$ refers to the well studied anisotropic XY spin-1/2 chain in a transverse field[25]. In this model, there exists an Ising transition line at $h = \pm J$ from the ferromagnetically ordered phase to a quantum paramagnetic phase. On the other hand, the vanishing of the gap at $\gamma = 0$ signifies another quantum phase transition belonging to a different universality class between two ferromagnetically ordered phases.

It can be established using a numerical diagonalization of the time-dependent Schrödinger equation involving the reduced Hamiltonian matrix that when the transverse field h or the alternating term δ is quenched as t/τ from $-\infty$ to ∞, so that the system crosses the quantum critical lines as shown in Fig. 1, the density of defects n in the final state satisfies the Kibble-Zurek prediction[22]. Our interest however lies in the generation of defects when the system is quenched along a gapless line. To achieve such a quenching, we vary the anisotropy parameter γ linearly as $\gamma = t/\tau$ from $-\infty$ to ∞, keeping h, δ and J fixed in such a way that the system always lies on the phase boundary $h^2 = \delta^2 + J^2$. In the limit $t \to -\infty$, γ is large and negative and hence in the ground state, the expectation value $\langle \sigma_j^x \sigma_{j+1}^x - \sigma_j^y \sigma_{j+1}^y \rangle = -1$. On the other hand, for an adiabatic evolution during the entire period of dynamics, this expectation value should be $+1$ in the final state. One can choose different critical lines for γ-quenching by choosing different values of h and δ on the critical surface.

The eigenvalue Λ_k^- given by Eq. (4) can be written as

$$
\begin{aligned}
\Lambda_k^- &= [(h - \sqrt{\delta^2 + J^2 \cos^2 k})^2 + 2h\sqrt{\delta^2 + J^2 \cos^2 k} \\
&+ \gamma^2 \sin^2 k - 2\sqrt{h^2 \delta^2 + h^2 J^2 \cos^2 k + \delta^2 \gamma^2 \sin^2 k}]^{1/2}.
\end{aligned}
\tag{6}
$$

On the gapless line $h^2 = \delta^2 + J^2$, the dispersion of the low-energy excitations at $k \to 0$ can be approximated as

$$
\Lambda_k^- = \sqrt{\frac{J^4 k^4}{4(\delta^2 + J^2)} + \frac{\gamma^2 J^2 k^2}{\delta^2 + J^2}} .
\tag{7}
$$

A close inspection of the above excitation spectrum suggests that when γ is quenched along the gapless line, Eq. (7) can be mapped to the spectrum of a 2×2 Landau-Zener (LZ) Hamiltonian[26,27] with two linearly approaching time-dependent levels. To show this explicitly, we note that in the limit of very slow quenching, $\tau \to \infty$, defects are produced by sets of modes between which the energy gap is very small. For the Hamiltonian in Eq. (3), this occurs in the region of $k = 0$ if we take $h^2 = \delta^2 + J^2$. Let us first set $\gamma = 0$ and $k = 0$. We then see that there are two modes, called $|I\rangle$ and $|II\rangle$, whose energies are zero; the other two modes have energies which are both far from zero and far from each other, and can therefore be ignored in a slow quenching calculation. The zero

energy modes are given by

$$
|I\rangle = \frac{1}{\sqrt{\delta^2 + (h+J)^2}} \begin{pmatrix} \delta \\ 0 \\ 0 \\ h+J \end{pmatrix},
$$

$$
|II\rangle = \frac{1}{\sqrt{\delta^2 + (h+J)^2}} \begin{pmatrix} 0 \\ \delta \\ h+J \\ 0 \end{pmatrix}.
\tag{8}
$$

We now deviate slightly from $k = 0$, still keeping $\gamma = 0$ in Eq. (3). Doing degenerate perturbation theory to first order in Jk^2, we find that the modes $|I\rangle$ and $|II\rangle$ remain eigenstates of the Hamiltonian, but their energies are now given by

$$
E_\pm = \pm \frac{J^2 k^2}{2\sqrt{\delta^2 + J^2}},
\tag{9}
$$

where we have used the relation $h^2 = \delta^2 + J^2$. Finally, we introduce the terms involving γ in Eq. (3). To first order in γ, we find that the Hamiltonian in the basis of $|I\rangle$ and $|II\rangle$ is given by

$$
h_k = \frac{1}{\sqrt{\delta^2 + J^2}} \begin{pmatrix} J^2 k^2/2 & -i\gamma Jk \\ i\gamma Jk & -J^2 k^2/2 \end{pmatrix}.
\tag{10}
$$

If we now perform a unitary transformation and vary γ in time, we see that the Hamiltonian is of the LZ form,

$$
h_k = \begin{bmatrix} \tilde{\gamma}(t)k & \tilde{J}^2 k^2/2 \\ \tilde{J}^2 k^2/2 & -\tilde{\gamma}(t)k \end{bmatrix},
\tag{11}
$$

where $\tilde{\gamma}$ and $\tilde{J}$ are renormalized parameters given by $\tilde{\gamma} = \gamma J/\sqrt{\delta^2 + J^2}$ and $\tilde{J}^2 = J^2/\sqrt{\delta^2 + J^2}$. The diagonal terms in Eq. (11) describe two time-dependent levels approaching each other linearly in time (since $\gamma = t/\tau$), while the minimum gap is given by the off-diagonal term $\tilde{J}^2 k^2/2$. The probability of excitations p_k from the ground state to the excited state for the k-th mode is given by the Landau-Zener transition formula[26,27]

$$
p_k = \exp \left[-\frac{2\pi \tilde{J}^4 k^4}{8k \frac{d\tilde{\gamma}(t)}{dt}} \right] = \exp \left[-\frac{\pi J^3 k^3 \tau}{4\sqrt{\delta^2 + J^2}} \right].
\tag{12}
$$

Note that for large τ, p_k is dominated by values of k close to 0. The density of excitations n in the final state and in the large τ limit is obtained by integrating over all modes in Eq. (12),

$$
n = \frac{2}{\pi} \int_0^{\pi/2} dk \, p_k \sim \frac{1}{\tau^{1/3}}.
\tag{13}
$$

The numerical integration of Eq. (13) for $\delta = J = 1$ is shown in Fig. 2 (a); although we have used the expression given in Eq. (12) for all values of k, the dominant contribution to n comes from the region near $k = 0$ where (12) can be trusted. This shows that when quenched along a gapless line, the density of defects in the final state exhibits a slower decay with τ as compared to the $1/\sqrt{\tau}$
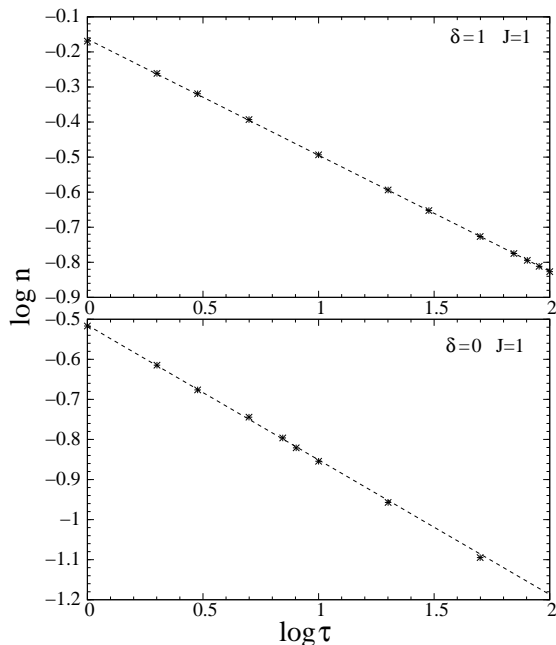
FIG. 2: Fig. 2 (a) shows the variation of defect density n with τ for $\delta = J = 1$ obtained by numerically integrating Eq. (13), using p_k given in Eq. (12). Fig. 2 (b) shows n obtained by direct numerical integration of the Schrödinger Equation with $\delta = 0$. In both figures, the fitted data shows a slope of $-1/3$ confirming the $n \sim 1/\tau^{1/3}$ behavior.

which is observed in the case when the gapless line is crossed by varying h[22]. Our study also reveals that although the spectrum of this system involves four levels, the quenching dynamics along the gapless line is essentially a two-level problem which can be studied using an effective Landau-Zener theory with parameters renormalized by δ.

The case $\delta = 0$ corresponds to quenching the system along the Ising critical line of the transverse XY model, and one obtains an identical scaling of the defect density. Fig. 2 (b) shows the $1/3$ power-law obtained by numerically solving the Schrödinger equation for the anisotropic transverse XY model when the anisotropy parameter γ is quenched along the gapless line $h = -J = 1$. (The Hamiltonian in Eq. (3) has a 2×2 block diagonal form if $\delta = 0$). One can also propose an alternative quenching scheme where the strength J is quenched from $-\infty$ to ∞, keeping h, δ and γ constant with $\delta^2 = h^2 + \gamma^2$. The duality relation discussed above leads to the conclusion that this quenching scheme is equivalent to the previous one with $\gamma = t/\tau$, and it yields a similar $1/\tau^{1/3}$ behavior.

The XY chain with an alternating transverse field can be further generalized by incorporating additional alternations in the strength or in the anisotropy of the interaction with the period of alternation being two. We denote the alternation in the strength and the anisotropy by J_s and γ_s, respectively, and for simplicity choose $J_s = 0$. Using a similar Jordan-Wigner transformation (Eq. (2))

followed by a Fourier transformation, we find an excitation spectrum of the form

$$
\begin{aligned}
\Lambda_k^{\pm} &= [h^2 + \delta^2 + (J^2 + \gamma_s^2)\cos^2 k + \gamma^2 \sin^2 k \\
&- 2\sqrt{(h^2 + \gamma_s^2 \cos^2 k)(\delta^2 + J^2 \cos^2 k) + \delta^2 \gamma^2 \sin^2 k}]^{1/2},
\end{aligned}
\tag{14}
$$

where the eigenvalue Λ_k^- has to be analyzed to explore the quenching dynamics. Eq. (14) shows that the role of the alternation γ_s is to renormalize the strength J; consequently, the phase boundary separating the paramagnetic and the ferromagnetic phase gets shifted to $h^2 = \delta^2 + J^2 - \gamma_s^2$, with arbitrary γ, at which the gapless excitations occur at $k = 0$. Quenching γ as t/τ along the new phase boundary $h^2 = \delta^2 + J^2 - \gamma_s^2$ with fixed values of J, h and γ_s, once again takes the system along a gapless line on a critical surface.

The dynamics can be reduced to a two-level problem as before, and the low-energy excitations above the gapless line are given by

$$
\Lambda_k^- = \sqrt{\frac{(J^2 - \gamma_s^2)^2 k^4}{4(\delta^2 + J^2)} + \frac{\gamma^2 J^2 k^2}{\delta^2 + J^2}} .
\tag{15}
$$

The defect density n in the final state decreases with τ as $1/\tau^{1/3}$ as can be derived from the Landau-Zener formula in an identical fashion. Similarly, one may set $\gamma_s = 0$ and $J_s \neq 0$ and consider an equivalent quenching scheme resulting in an identical scaling of the defect density.

The behavior of the defect density when quenched along a gapless line suggests the following general scaling relation of the defect density for a d-dimensional quantum system. Let the excitations on the gapless quantum critical line be of the form $\omega_{\vec{k}} \sim \alpha |\vec{k}|^z$, where z is the dynamical exponent and the parameter $\alpha = t/\tau$ is quenched from $-\infty$ to ∞. Using a perturbative method involving the Fermi Golden rule along with the fact that the system is initially prepared in the ground state, the defect density can be approximated as[18]

$$
n \simeq \int \frac{d^d k}{(2\pi)^d} \left| \int_{-\infty}^{\infty} d\alpha \, \langle \vec{k} | \frac{\partial}{\partial \alpha} | 0 \rangle \, e^{i\tau \int^{\alpha} \delta\omega_{\vec{k}}(\alpha')d\alpha'} \right|^2.
\tag{16}
$$

Assuming a general scaling form of the instantaneous excitation $\delta\omega_{\vec{k}}(\alpha') = k^a f(\frac{\alpha k^z}{k^a})$, where $k = |\vec{k}|$, and k^a denotes the higher order term in the excitation spectrum on the gapless line. Defining a new variable $\xi = \alpha k^{z-a}$, we obtain the scaling behavior of the defect density as

$$
n \sim 1/\tau^{d/(2a-z)} .
\tag{17}
$$

The case $d = 1$, $a = 2$ and $z = 1$ has been discussed in the present work. Note that the correlation length exponent ν does not appear in the expression in Eq. (17) because our quench dynamics always keeps the system on a critical line.

III. CONCLUSION

In conclusion, we have studied the defect density produced in the final state when a generalized spin-1/2 XY chain with an alternating transverse field as well as an alternating nearest-neighbor interaction is quenched along the Ising critical line by varying the anisotropy parameter γ. We show that the non-adiabatic transition probability and hence the defect density can be estimated using an equivalent Landau-Zener problem in which the parameters are renormalized by the alternating parameters δ and γ_s (or J_s). We find that the defect density decays with the characteristic time scale of quenching given by $1/\tau^{1/3}$. The defect scaling exponent obtained here does not fit the KZ scaling $1/\tau^{d\nu/(z\nu+1)}$. In the present quenching scheme, the system is always on a critical surface, and therefore the critical exponent ν does not appear in the scaling of the defect density. The quenching scheme used here is different from the other quenching schemes through a gapless phase[18,19,20] where the system starts from a non-critical (gapped) point, goes through a critical (gapless) point or critical surface, and eventually ends again at a non-critical point.

Acknowledgments

* Electronic address: udiva@iitk.ac.in
† Electronic address: dutta@iitk.ac.in
‡ Electronic address: diptiman@cts.iisc.ernet.in

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