

Tuning the presence of dynamical phase transitions in a generalized XY spin chain

Uma Divakaran,¹ Shraddha Sharma,² and Amit Dutta²

¹*UM-DAE Center for Excellence in Basic Sciences, Mumbai 400 098, India*

²*Department of Physics, Indian Institute of Technology, Kanpur 208 016, India*

We study an integrable spin chain with three spin interactions and the staggered field (λ) while the latter is quenched either slowly (in a linear fashion in time (t) as t/τ where t goes from a large negative value to a large positive value and τ is the inverse rate of quenching) or suddenly. In the process, the system crosses quantum critical points and gapless phases. We address the question whether there exist non-analyticities (known as dynamical phase transitions (DPTs)) in the subsequent real time evolution of the state (reached following the quench) governed by the final time-independent Hamiltonian. In the case of sufficiently slow quenching (when τ exceeds a critical value τ_1), we show that DPTs, of the form similar to those occurring for quenching across an isolated critical point, can occur even when the system is slowly driven across more than one critical point and gapless phases. More interestingly, in the anisotropic situation we show that DPTs can completely disappear for some values of the anisotropy term (γ) and τ , thereby establishing the existence of boundaries in the $(\gamma - \tau)$ plane between the DPT and no-DPT regions in both isotropic and anisotropic cases. Our study therefore leads to a unique situation when DPTs may not occur even when an integrable model is slowly ramped across a QCP. On the other hand, considering sudden quenches from an initial value λ_i to a final value λ_f , we show that the condition for the presence of DPTs is governed by relations involving λ_i , λ_f and γ and the spin chain must be swept across $\lambda = 0$ for DPTs to occur.

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I. INTRODUCTION

Inspired by the concept of non-analyticities associated with the free-energy density of a classical system at a finite temperature transition marked by the zeroes of the partition function in a complex temperature plane¹ (see also [2,3]), recently there has been a proposal of quantum dynamical phase transitions (DPTs) in a quenched quantum many-body system⁴. Associated non-analyticities are quantified in terms of the overlap amplitude or the Loschmidt overlap (LO) defined for the quenched quantum system. Focussing on the sudden quenching case and denoting the ground state of the initial Hamiltonian as $|\psi_0\rangle$, the Loschmidt overlap is defined as $G(t) = \langle \psi_0 | e^{-iH_f t} | \psi_0 \rangle$; here, H_f is the final Hamiltonian reached after the quenching process. DPTs occur when the initial state is orthogonal to the evolved state and the LO vanishes. Generalizing $G(t)$ to $G(z)$ defined in the complex time (z) plane, one introduces the corresponding dynamical free energy density, $f(z) = -\lim_{L \rightarrow \infty} \log G(z)/L^d$, where L is the linear dimension of a d -dimensional system. One then looks for the zeros of the $G(z)$ (or non-analyticities in $f(z)$), known as Fisher zeros. For a transverse Ising chain, it has been observed⁴ that when the system is suddenly quenched across the quantum critical point (QCP)^{5,6}, the lines of Fisher zeros cross the imaginary time axis at instants of real time t^* ; at these instants the rate function of the return probability defined as $I(t) = -\log |G(t)|^2/L$ shows sharp non-analyticities signaling the occurrence of DPTs.

The initial observation by Heyl *et al.*⁴ that DPTs are associated with the sudden quenches across the QCP has been verified in several studies^{7-10,16}. However, subse-

quently it has been shown that DPTs are not necessarily connected with the passage through the equilibrium QCP and may occur following a sudden quench even within the same phase (i.e., not crossing the QCP) for both integrable¹¹ as well as non-integrable models¹². Subsequently, these studies have been generalized to two-dimensional systems^{13,14} and the role of topology¹³ and the dynamical topological order parameter have been investigated¹⁵. We note in the passing that the rate function $I(t)$ is related to the Loschmidt echo which has been studied in the context of decoherence¹⁷⁻²⁶ and the work-statistics^{27,28}. The finite temperature counterpart of the Loschmidt echo²⁹, namely the characteristic function has also been useful in studies of the entropy generation and emergent thermodynamics in quenched quantum systems^{30,31}. In fact, the rate function (of the return probability) discussed above in the context of DPTs can be connected to the work distribution function corresponding to the zero work in a double quenching experiment⁴.

The periodic occurrences of non-analyticities in the rate function for an integrable model was first reported in the context of a slow quenching of the transverse field in the transverse Ising chain across its QCPs³². Very recently, associated DPTs have also been related to Fisher zeros crossing the imaginary axis of the complex time plane³³. This is believed to be in general true for an integrable model reducible to decoupled two level problems quenched slowly across its QCP.

In this paper, we extend the previous studies further to the slow as well as sudden quenching of an integrable quantum Ising model with complicated interactions across the QCPs (and also gapless phases) and establish that DPTs may completely disappear in some sit-

uations depending on the quenching rate (or amplitude in sudden quench) and system parameters. This is an observation that, to the best of our knowledge, has not been reported earlier particularly for the slow quenching. We note at the outset that for the slow quenching, the final state is prepared through the variation of a parameter of the Hamiltonian as t/τ across the QCP to the final value of time (and hence, of the parameter); on the contrary, for the sudden quenches the final state happens to be the ground state of the initial Hamiltonian. In both the cases $G(t)$ describes the subsequent temporal evolution of the system with the final time-independent Hamiltonian setting the origin of time ($t = 0$) immediately after the quenching is complete. Let us also note that the numerical calculations are performed for a finite system, hence Fisher zeros do not coalesce into a line, rather constitute a set of closely spaced points.

We would also like to mention that the slow quenching dynamics across or to a QCP has been studied in the context of possible Kibble-Zurek (KZ) scaling^{34,35} of the defect density and the residual energy^{36,37} which have been explored in various situations^{38–48}. (For reviews, see [49–51].)

The paper is organized in the following manner: In Sec. II, we introduce the connection between Fisher zeros, DPTs and the slow (as well as sudden) quenching of a generic two-level integrable model. In Sec. III, on the other hand, we introduce a specific model, namely, a generalized transverse Ising chain with three-spin interactions and a staggered magnetic field (λ) and present its phase diagram. In Sec. IV we focus on slow quenches and show that DPTs always occur in the isotropic situation even when the system is quenched across two critical points and gapless phases if the quenching is not too rapid. On the contrary, in the anisotropic case, there is a clear boundary separating the DPT and the no-DPT region; this establishes that the slow quenching of an integrable model across its QCP does not necessarily lead to DPTs. Finally, in Sec. V, we consider the sudden quenching of the staggered field and show how the presence of DPTs following the quench is dictated by relations involving the initial and final values of the field and the anisotropy parameter; it is worth mentioning that the spin chain must be quenched across $\lambda = 0$ for DPTs to occur.

II. QUENCHES OF AN INTEGRABLE MODEL AND DPT

Let us consider an integrable model reducible to a two level system for each momenta mode; the system is initially ($t \rightarrow -\infty$) in the ground state $|1_k^i\rangle$ of the initial Hamiltonian for each mode. We first consider the slow quenching case. The Hamiltonian is characterized by a parameter λ which is quenched from an initial value λ_i following the quenching protocol $\lambda(t) = t/\tau$ to a final value λ_f so chosen that the system crosses the QCP

at $\lambda = \lambda_c$ in the process. Since the condition for an adiabatic dynamics breaks in the vicinity of the QCP, one arrives at a final state (for the k -th mode) given by $|\psi_{f_k}\rangle = v_k|1_k^f\rangle + u_k|2_k^f\rangle$, with $|u_k|^2 + |v_k|^2 = 1$; here, $|1_k^f\rangle$ and $|2_k^f\rangle$ are the ground state and the excited states of the final Hamiltonian $H_{f_k}(\lambda_f)$ with corresponding energy eigenvalues $\epsilon_{k,1}^f$ and $\epsilon_{k,2}^f$, respectively. One can define the LO for the mode k as $\langle\psi_{f_k}|\exp(-H_{f_k}z)|\psi_{f_k}\rangle$ and the corresponding dynamical free energy⁴, $f_k(z) = -\log\langle\psi_{f_k}|\exp(-H_{f_k}z)|\psi_{f_k}\rangle/L$, where z is the complex time with $z = R + it$, R being the real part and t the imaginary part. Summing over the contributions from all the momenta modes and converting summation to the integral in the thermodynamic limit, one gets

$$\begin{aligned} f(z) &= -\int_0^\pi \frac{dk}{2\pi} \log\left(|v_k|^2 \exp(-\epsilon_{k,1}^f z) + |u_k|^2 \exp(-\epsilon_{k,2}^f z)\right) \\ &= -\int_0^\pi \frac{dk}{2\pi} \log\left((1-p_k) \exp(-\epsilon_{k,1}^f z) + p_k \exp(-\epsilon_{k,2}^f z)\right); \end{aligned} \quad (1)$$

we reiterate that t is measured from the instant the final state $|\psi_{f_k}\rangle$ is reached after the slow quench.

We then immediately find the zeros (i.e., the Fisher zeros) of the “effective” partition function where $f(z)$ is non-analytic as

$$z_n(k) = \frac{1}{(\epsilon_{k,2}^f - \epsilon_{k,1}^f)} \left(\log\left(\frac{p_k}{1-p_k}\right) + i\pi(2n+1) \right), \quad (2)$$

where $n = 0, \pm 1, \pm 2, \dots$. It is to be emphasized that in $z_n(k)$, the difference of the eigenvalues ($\epsilon_{k,2}^f - \epsilon_{k,1}^f$), rather than the eigenvalues themselves, appear. The Fisher zeros constitute a line (more precisely, closely spaced points) corresponding to each n in the complex z plane³³. The critical mode k_c (for which the gap in the spectrum vanishes at $\lambda = \lambda_c$) remains frozen in the initial state and hence $p_{k=k_c} = 1$, while for modes far away from the critical mode $p_k \rightarrow 0$; therefore $z_n(k)$'s goes from $-\infty$ to ∞ in the thermodynamic limit as k changes. From the continuity argument, there must exist one specific value of k_* for which $p_{k=k_*} = 1/2$ and $\text{Re}(z_n(k)|_{k=k_*})$ vanishes; from (2) we note that the lines of Fisher zeros cross the imaginary axis for k_* .

The rate function of the return probability in this case can be evaluated exactly in the form^{32,33}

$$\begin{aligned} I(t) &= -\frac{\log|G(t)|^2}{L} = 2 \text{Re}f(z) \\ &= -\int_0^\pi \frac{dk}{2\pi} \log\left(1 + 4p_k(p_k - 1) \sin^2 \frac{(\epsilon_{k,2}^f - \epsilon_{k,1}^f)t}{2}\right); \end{aligned} \quad (3)$$

the non-analyticities in $I(t)$ appear at the values of the real time t_n^* s given by

$$t_n^* = \frac{\pi}{(\epsilon_{k_*,2}^f - \epsilon_{k_*,1}^f)} (2n+1) \quad (4)$$

derived by setting $\text{Re}(z_n(k_*)) = 0$ in Eq. (2) as the argument of the logarithm in Eq. (3) vanishes for $k = k_*$ when $p_{k=k_*} = 1/2$. Again, the time instants t_n^* depend on $(\epsilon_{k_*,2}^f - \epsilon_{k_*,1}^f)$. However, for the case $\epsilon_{k,2}^f = -\epsilon_{k,1}^f = \epsilon_k^f$, Eq. (4) gets simplified to

$$t_n^* = \frac{\pi}{\epsilon_{k_*}^f} \left(n + \frac{1}{2} \right) \quad (5)$$

We shall briefly dwell on the case of sudden quenching⁴ when the parameter λ is suddenly changed from an initial value λ_i to a final value λ_f ; in this case, the final state $|\psi_{k_f}\rangle$ is the initial ground state $|1_k^i\rangle$ (corresponding to λ_i) while the Hamiltonian gets modified to the final Hamiltonian; the LO for the given mode is given by $\langle 1_k^i | \exp(-H_{f_k} z) | 1_k^i \rangle$. Following a similar line of arguments as above, one can show that the dynamical free energy has a similar form as in Eq. (1) with $|u_k|^2 \rightarrow |\tilde{u}_k|^2 = \tilde{p}_k = |\langle 1_k^i | 2_k^f \rangle|^2$ and $|v_k|^2 \rightarrow |\tilde{v}_k|^2 = |\langle 1_k^i | 1_k^f \rangle|^2$. Therefore, one finds a similar expression for the rate function in Eq. (3) (with $p_k \rightarrow \tilde{p}_k$) which shows non-analyticities at the instants of real time again given by Eq. (4) when $\tilde{p}_{k=k_*} = 1/2$.

III. GENERALIZED SPIN MODEL

In this section, we shall consider a generalized spin-1/2 quantum XY chain with a two sublattice structure in the presence of a three spin interaction ($J_3 > 0$) and a staggered field (h) described by the Hamiltonian

$$\begin{aligned} H = & -h \sum_i (\sigma_{i,1}^z - \sigma_{i,2}^z) - J_1 \sum_i (\sigma_{i,1}^x \sigma_{i,2}^x + \sigma_{i,1}^y \sigma_{i,2}^y) \\ & - J_2 \sum_i (\sigma_{i,2}^x \sigma_{i+1,1}^x + \sigma_{i,2}^y \sigma_{i+1,1}^y) - J_3 \sum_i (\sigma_{i,1}^x \sigma_{i,2}^z \sigma_{i+1,1}^x \\ & + \sigma_{i,1}^y \sigma_{i,2}^z \sigma_{i+1,1}^y) - J_3 \sum_i (\sigma_{i,2}^x \sigma_{i+1,1}^z \sigma_{i+1,2}^x \\ & + \sigma_{i,2}^y \sigma_{i+1,1}^z \sigma_{i+1,2}^y), \end{aligned} \quad (6)$$

where i is the site index and the additional subscript 1(2) defines the odd (even) sublattice. The parameter J_1 describes the XY interaction between the spins on sublattice 1 and 2 while J_2 describes the XY interaction between spins on sublattice 2 and 1 such that J_1 is not necessarily equal to J_2 . In spite of the complicated nature of interactions, this spin chain is integrable and exactly solvable in terms of a pair of Jordan-Wigner fermions^{52,53} defined on even and odd sublattices as $\sigma_{i,1}^+ = \left[\prod_{j<i} (-\sigma_{j,1}^z)(-\sigma_{j,2}^z) \right] a_i^\dagger$, and $\sigma_{i,2}^+ = \left[\prod_{j<i} (-\sigma_{j,1}^z)(-\sigma_{j,2}^z)(-\sigma_{i,1}^z) \right] b_i^\dagger$, where $\sigma_{i,1}^z = 2a_i^\dagger a_i - 1$ and $\sigma_{i,2}^z = 2b_i^\dagger b_i - 1$. The Fermion operators a_i and b_i can be shown to satisfy fermionic anticommutation relations.

In the k -space, the reduced Hamiltonian is given by

$$H_k = \alpha \cos k \hat{1} - \frac{1}{2} \begin{bmatrix} \lambda & -(1 + \gamma e^{-ik}) \\ -(1 + \gamma e^{+ik}) & -\lambda \end{bmatrix}, \quad (7)$$

where $\lambda = h/J_1$, $\alpha = J_3/J_1$ and $\gamma = J_2/J_1$ and $\hat{1}$ is the 2×2 identity operator and the second part represents the 2×2 Landau-Zener (LZ) part of the Hamiltonian; we shall also use the notation $\Delta_k = (1 + \gamma e^{-ik})$ below. The corresponding eigenvalues of the reduced Hamiltonian H_k are

$$\begin{aligned} \tilde{\epsilon}_k^\pm &= \alpha \cos k \pm \epsilon_k \\ &= \alpha \cos k \pm \frac{1}{2} \sqrt{\lambda^2 + \gamma^2 + 1 + 2\gamma \cos k}. \end{aligned} \quad (8)$$

The phase diagram obtained by analyzing the spectrum in Eq. (8) is shown in Fig. 1. We shall consider the slow as well as sudden quenching dynamics of the Hamiltonian (6) by varying the parameter λ across the QCPs and gapless phases and probe the corresponding DPT scenario.

As evident from Eq. (7), the term $\alpha \cos k$ leads to the rich phase diagram of the model under consideration by introducing gapless phases of different kinds, where the gap in the spectrum vanishes solely due to the presence of $\alpha \cos k$. However, this term does not participate in the dynamics. This is because of the fact that the term $\alpha \cos k$ is associated with the identity operator which always commutes with the time evolution operator for any type of temporal evolution. The dynamics of the system is, therefore, entirely determined by the LZ part of Eq. (7). This in fact leads to a conspicuous behavior as far as DPTs are concerned as we shall discuss below; furthermore, only the terms $\pm \epsilon_k$ appearing in the eigenvalues in Eq. (8) determine the instants at which DPTs occur. This is also clear from Eq. (4) that only the difference of eigenvalues plays a role in determining t_n^* s and hence, the results will be completely independent of the parameter α . Additionally, the eigenfunctions of the Hamiltonian H_k are also identical to those of the LZ part.

We note in the passing that Hamiltonian of the form (6) has been studied extensively^{54,55} over decades; recently topological aspects of this kind of models have also been explored⁵⁶.

IV. SLOW QUENCHES: DPT-NO DPT BOUNDARY

Let us consider a variation of the field $\lambda = t/\tau$ from -10 to $+10$ so that the system is quenched from one antiferromagnetic phase to the other crossing both the gapless phases. The probability of excitations p_k following the quench is given by the LZ transition probability^{57,58} $p_k = e^{-\frac{\pi |\Delta_k|^2 \tau}{2}}$; here, $|\Delta_k|^2 = (1 + \gamma^2 + 2\gamma \cos k)$ which vanishes for $k = \pi$ at the boundary between the antiferromagnetic (AF) phase and the gapless phase GPI for

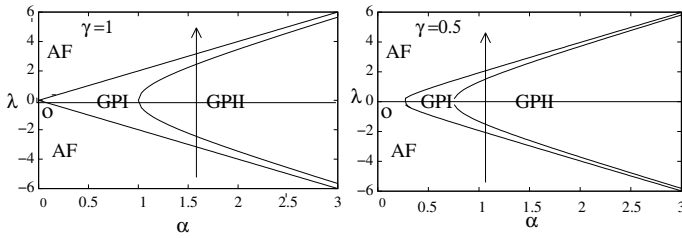


FIG. 1: (Color online) Phase diagram of the Hamiltonian (7) in the $\alpha - \lambda$ plane for the isotropic case $\gamma = 1$ and an anisotropic case with $\gamma = 0.5$. For a fixed α and a large magnitude of the rescaled field λ , the spin chain is anti-ferromagnetic (AF). On the other hand, when λ is reduced, the system undergoes quantum phase transitions from Gapless-I (GPI) phase characterized by two Fermi points to the Gapless-II (GPII) phase characterized by four Fermi points. The vertical line shows the direction of quenching. After [59].

the isotropic case $\gamma = 1$. Probing the rate function, we indeed find periodic occurrences of sharp non-analyticities as expected in the case of quenching across an isolated QCP (see the numerical result presented in the top panel of Fig. (2)); the instants at which these non-analyticities appear can be matched with those obtained from Eq. (5) with $\epsilon_{k_*}^f = (1/2)\sqrt{\lambda_f^2 + 1 + \gamma^2 + 2\gamma \cos k_*} = (1/2)\sqrt{\lambda_f^2 + 2 + 2\cos k_*}$ for $\gamma = 1$, λ_f being the final parameter value reached after the quenching. It is worth mentioning that the dynamics is completely insensitive to the fact that the system is driven across gapless phases in the process of quenching and hence no trace of gapless phases is reflected in DPTs. However, the occurrences of DPTs also require the condition that the minimum value of the non-adiabatic transition probability $p_{k=0} = \exp(-\frac{\pi|\Delta_k|^2|_{k=0}\tau}{2}) = \exp(-\frac{\pi(1+\gamma)^2\tau}{2})$ must be less than $1/2$ so that a k_* (for which $p_{k=k_*} = 1/2$) exists. This does not happen if the quenching is too rapid, i.e., $\tau < \tau_1(\gamma) = 2 \log 2 / \{\pi(1 + \gamma)^2\}$ for $\gamma \neq 1$; for $\gamma = 1$, $\tau_1(\gamma)|_{\gamma \rightarrow 1} = \log 2 / (2\pi)$. One therefore does not indeed observe DPTs even in the isotropic case for too rapid quenching processes.

We now move to the more interesting situation which arises in the anisotropic case ($\gamma \neq 1$); in this case, $|\Delta_k|^2 = (1 + \gamma^2 + 2\gamma \cos k)$, assumes the minimum value at the boundary between AF and the GPI phase for the mode $k = \pi$ and is given by $|\Delta_k|^2 = (1 - \gamma)^2$, and hence the maximum value of the non-adiabatic transition probability $p_k^{\max} = \exp(-\pi(1 - \gamma)^2\tau/2)$. As emphasized before, DPTs can occur only when $p_k = 1/2$. If the maximum possible value of p_k^{\max} is less than $1/2$, no DPT can appear even when the system is quenched across the QCPs and gapless phases. We therefore find a boundary

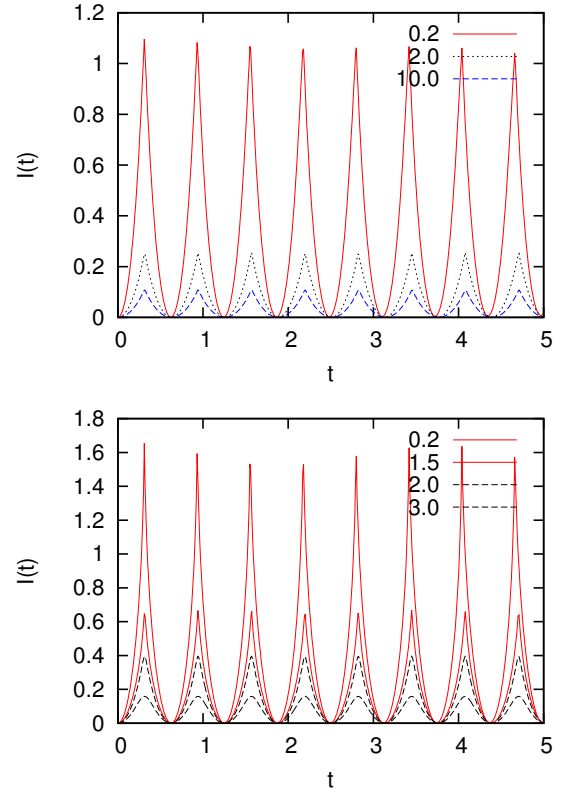


FIG. 2: (Color online) The rate function $I(t)$ as obtained by numerically integrating Eq. (3) is plotted as a function of time following a slow quench from a large negative to a large positive value of λ . The upper panel corresponds to $\gamma = 1$ where there are periodic occurrences of DPTs for all values of τ 's subject to the condition that $p_{k|_{k=0}} < 0.5$, i.e., $\tau > \tau_1(\gamma)|_{\gamma \rightarrow 1}$. The lower panel corresponds to the anisotropic situation with $\gamma = 0.5$ for which the critical value of τ as obtained from Eq. (9) is given by $\tau_2(\gamma) = 1.76$. As discussed in the text, the figure shows the presence of DPTs for $\tau_1(\gamma) < \tau < \tau_2(\gamma)$ while they disappear for $\tau > \tau_2(\gamma)$. The red colored (solid line) show sharp peaks whereas black colored (dashed lines) show peaks those are rounded off.

in the $(\gamma - \tau)$ plane given by the equation:

$$\exp(-\pi(1 - \gamma)^2\tau/2) = 1/2; \quad \tau = \tau_2(\gamma) = \frac{2 \log 2}{\pi(1 - \gamma)^2}. \quad (9)$$

For a fixed γ , if τ exceeds $\tau_2(\gamma)$, DPTs disappear. This is verified numerically and shown in the lower panel of Fig. (2) where we evaluate $I(t)$ by numerically calculating p_k s and using the values $\epsilon_{1,k}^f = -\epsilon_k^f$ and $\epsilon_{2,k}^f = \epsilon_k^f$ (corresponding to $\lambda_f = 10$) in Eq. (3); we show that DPTs occurring for $\tau < \tau_2(\gamma)$, disappear when τ exceeds $\tau_2(\gamma)$. Referring to the situation $\gamma = 1$, when maximum value of p_k (for $k = \pi$) is equal to unity and $\tau_2(\gamma)|_{\gamma \rightarrow 1} \rightarrow \infty$, DPTs periodically appear for all values of $\tau > \tau_1(\gamma)$. On the contrary, for $\gamma \neq 1$, there always exists a critical $\tau_2(\gamma)$ and DPTs appear only when $\tau_1(\gamma) < \tau < \tau_2(\gamma)$. We would like to emphasize that these observations are

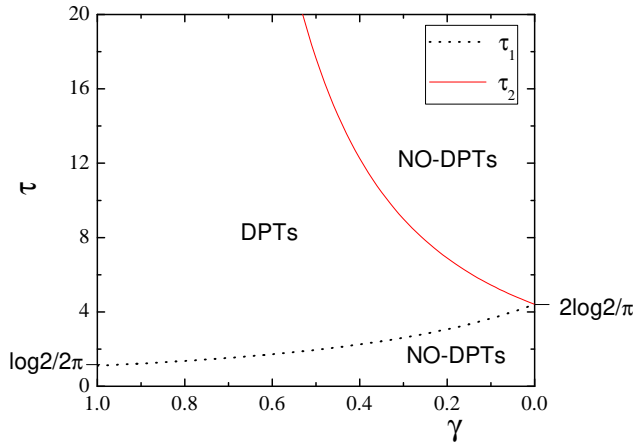


FIG. 3: (Color online) The phase diagram in the γ - τ plane showing the DPT and the no-DPT regions following a slow quench as discussed in Fig. 2. The upper curve corresponds to the condition presented in Eq. (9) i.e., $\tau_2(\gamma) = 2 \log 2 / (\pi(1 - \gamma)^2)$ which diverges in the isotropic case ($\gamma = 1$). The lower curve denoted by $\tau_1(\gamma) = 2 \log 2 / (\pi(1 + \gamma)^2)$ is obtained from the requirement $p_{k=0} = 1/2$ and $\tau_1(\gamma)|_{\gamma=1} = \log 2 / 2\pi$. It is to be noted that the values of both $\tau_1(\gamma)$ and $\tau_2(\gamma)$ are zoomed by a factor of 10 for better visibility. The DPTs exist for the γ and τ values lying in the region bounded by $\tau_2(\gamma)$ and $\tau_1(\gamma)$.

all appropriately supported by the behavior of the lines of Fisher zeros, e.g., one can verify that the lines of Fisher zeros never cross the imaginary axis in the no-DPT region in the anisotropic case. All these conditions are summarised in Fig. (3).

Interestingly, the relation (9) does not depend on the parameter α which plays no role in the temporal evolution of the system. Let us also address the question what happens when α is quenched keeping λ and γ fixed. In this case, the initial ground state of the Hamiltonian H_k (which is also the ground state of the LZ part of the Hamiltonian (7)), only evolves through an overall phase accumulation. Since the LZ part is unaltered, there is no non-trivial dynamics for any mode k , and hence, no DPT is expected; the evolved state is never orthogonal to the state $|\psi_{f_k}\rangle$. It is also noteworthy that in the anisotropic case, the defect density shows an exponential decay (with τ) as opposed to the standard power-law KZ scaling⁵⁹.

V. SUDDEN QUENCHES: CONDITIONS FOR DPTS

In this section, we shall consider a sudden quenching of the parameter λ from an initial λ_i to a final value λ_f . We address the questions whether DPTs are always present in the subsequent temporal evolution and how does the situation get altered in the anisotropic case in comparison to the isotropic case. Remarkably, as we shall illustrate

below, in this case also whether DPTs are present or absent depend on some conditions involving λ and γ both for $\gamma = 1$ and $\neq 1$. Referring to the Hamiltonian (7), we find that the ground state and the excited state, i.e., the adiabatic basis states, for a given λ (say, λ_i) is given by

$$\begin{aligned} |1_k^i\rangle &= \cos \frac{\theta_k}{2} (1, 0)^T - \sin \frac{\theta_k}{2} (0, 1)^T \\ |2_k^i\rangle &= \sin \frac{\theta_k}{2} (1, 0)^T + \cos \frac{\theta_k}{2} (0, 1)^T, \end{aligned} \quad (10)$$

where $\tan \theta_k = -|\Delta_k|/\lambda$ which clearly does not depend on α . When the field λ_i is suddenly changed to λ_f , the excitation probability is given by $\tilde{p}_k = |\tilde{u}_k|^2 = |\langle 1_k^i | 2_k^f \rangle|^2$. As discussed before, the necessary condition for the presence of a DPT requires $\tilde{p}_k|_{k=k_*} = 1/2$. Using Eqs. (10), we immediately find

$$\begin{aligned} \tilde{p}_k &= |\tilde{u}_k|^2 = |\langle 1_k(\lambda_i) | 2_k(\lambda_f) \rangle|^2 = \sin^2[(\theta_k^i - \theta_k^f)]/2 \\ &= \frac{1}{2} \left[1 - \frac{\lambda_f \lambda_i + |\Delta_k|^2}{\sqrt{(\lambda_i^2 + |\Delta_k|^2)} \sqrt{(\lambda_f^2 + |\Delta_k|^2)}} \right]; \end{aligned} \quad (11)$$

it should be noted that \tilde{p}_k depends on λ_i , λ_f and γ but never on α .

To predict the presence of DPTs, it is sufficient to analyze $|\tilde{u}_{k=0}|^2$ and $|\tilde{u}_{k=\pi}|^2$; the necessary condition for DPT would then be $|\tilde{u}_{k=\pi}|^2 > 1/2$ and $|\tilde{u}_{k=0}|^2 < 1/2$. (We recall that the $|\Delta_k|$ is minimum for the mode $k = \pi$ and hence probability of excitation is maximum for that particular mode). If these conditions are satisfied, from the argument of continuity one concludes that there must exist a k_* for which $|\tilde{u}_{k=k_*}|^2 = 1/2$, ensuring the existence of DPTs. We note that this is the most generic condition for DPTs to occur as long as one can sharply define a k_* . Using (11), one can show that \tilde{p}_k becomes equal to 1/2 for a mode k only when

$$\frac{\lambda_f \lambda_i + |\Delta_k|^2}{\sqrt{(\lambda_i^2 + |\Delta_k|^2)} \sqrt{(\lambda_f^2 + |\Delta_k|^2)}} = 0 \implies \lambda_f \lambda_i + |\Delta_k|^2 = 0, \quad (12)$$

where $|\Delta_k|^2$ is evaluated at the corresponding value of k . To illustrate the main point in a transparent manner, we choose $\lambda_f = -\lambda_i = \lambda$, (or the other way round i.e., $\lambda_f = -\lambda_i = -\lambda$) for which Eq. (12) assumes a simpler form:

$$\frac{-\lambda^2 + |\Delta_k|^2}{\sqrt{(\lambda^2 + |\Delta_k|^2)} \sqrt{(\lambda^2 + |\Delta_k|^2)}} = 0 \implies \lambda^2 = |\Delta_k|^2. \quad (13)$$

We shall analyze the condition given in (13) for the modes $k = 0$ and $k = \pi$ for both $\gamma = 1$ and $\gamma \neq 1$. In the former case ($\gamma = 1$), $|\tilde{u}_{k=\pi}|^2 = 1$ as the off-diagonal terms of the LZ part of the Hamiltonian (7) vanish for $k = \pi$ so that this mode is temporally frozen. On the other hand, the condition that $|\tilde{u}_{k=0}|^2 \leq 1/2$, demands $\lambda^2 \leq \Delta_k^2|_{k=0} = 4$. This implies that whenever the field

λ is quenched from a value $\lambda_i \geq -2$ to a final value $\lambda_f \leq +2$, DPTs will indeed appear. Otherwise, they are absent. This is numerically verified as shown in the upper panel of Fig. 4.

Proceeding to the anisotropic case, we find from Eq. (13) that the condition $|\tilde{u}_{k=\pi}|^2 \geq 1/2$, leads to $\lambda \geq (1 - \gamma)$ while the requirement $|\tilde{u}_{k=0}|^2 \leq 1/2$, yields $\lambda \leq (1 + \gamma)$. Therefore, for a sudden quench from $-\lambda$ to $+\lambda$ with a given γ , one finds a range of λ dictated by the condition $(1 - \gamma) \leq \lambda \leq (1 + \gamma)$ for which DPTs would appear as numerically verified in the lower panel of Fig. 4. This condition immediately reduces to the isotropic case for $\gamma = 1$, where, as shown above, the magnitude of λ should be less than 2 to observe DPTs.

Referring to Eq. (12), we find that for DPTs to occur, the quantity $\lambda_i \lambda_f$ must be negative; that implies that the spin chain must be quenched across $\lambda = 0$. In that sense, the line $\lambda = 0$ is special; this is in congruence with the observation reported in the Ref. [60] where it has been shown the Loschmidt echo when studied as a function of λ shows a dip only at $\lambda = 0$, thereby detecting only a special point of the phase diagram. Therefore for a generic situation, the condition for DPT to occur would be $|\tilde{u}_{k=\pi}|^2 > 1/2$ and $|\tilde{u}_{k=0}|^2 < 1/2$ along with the condition the system is quenched across $\lambda = 0$; for a quench from an initial value $-\lambda_i$ to a final value λ_f , Eq. (12) then leads to a more generic condition $(1 - \gamma) \leq \sqrt{|\lambda_i| \lambda_f} \leq (1 + \gamma)$ for DPTs to occur. (If the quenching is from $+\lambda_i$ to $-\lambda_f$, the condition gets modified to $(1 - \gamma) \leq \sqrt{|\lambda_i| |\lambda_f|} \leq (1 + \gamma)$.) This has also been numerically verified. What needs to be emphasized is that whether DPTs are present following a sudden quench is completely independent of the fact whether the system is quenched across a QCP or not; therefore, the passage through a QCP is never a necessary criteria. All these above conditions are summarized in Fig. 5

VI. CONCLUSION

We have explored the possibility of DPTs following slow as well as sudden quenches of a model Hamiltonian with a rich phase diagram with two gapless phases. We find some worth mentioning results not reported before. The term of the reduced Hamiltonian that results into these gapless phases do not participate in the dynamics and hence the passage through the gapless phases is not reflected in the behavior of DPTs those may occur following the quench both in isotropic and anisotropic cases. Consequently, for the slow quenches in the isotropic case, there are periodic occurrences of DPTs as expected in the case of a slow passage of an integrable model through an isolated QCP if the quenching is not too rapid (i.e., for $\tau > \tau_1(\gamma)$). On the contrary, in the anisotropic case, one finds a region in which DPTs exist bounded by two limiting quenching rates $\tau_1(\gamma)$ and $\tau_2(\gamma)$ in the $\gamma - \tau$ plane as summarized in Fig. 3; in the isotropic case $\tau_1(\gamma)|_{\gamma \rightarrow 1} = 2 \log 2/\pi$ and $\tau_2(\gamma)|_{\gamma \rightarrow 1} \rightarrow \infty$. This model

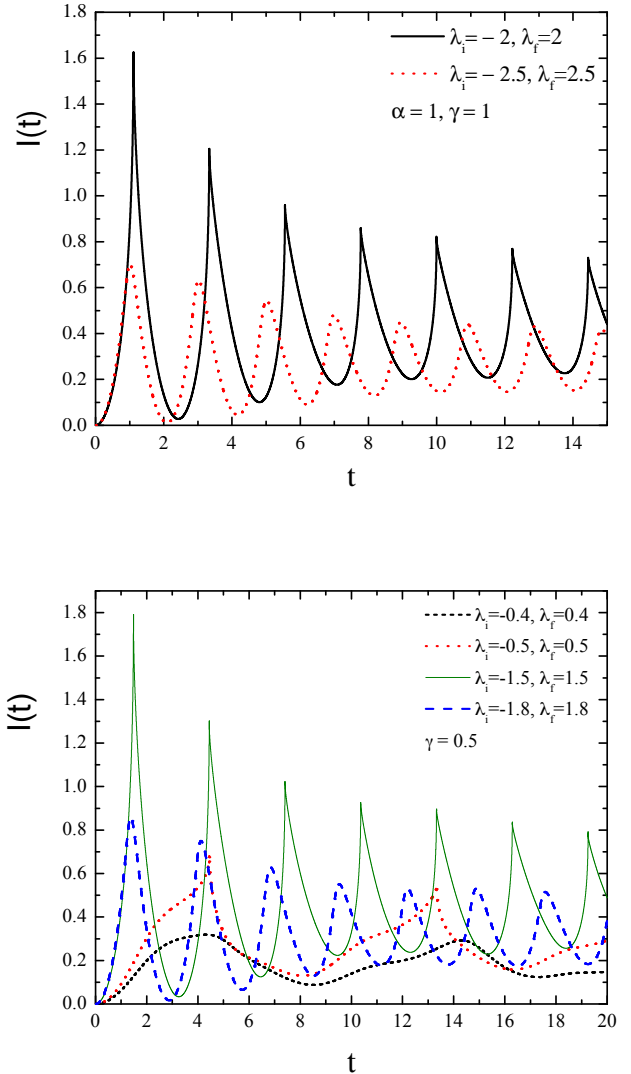


FIG. 4: (Color online) The presence and the absence of DPTs following a sudden quenching of λ from $-\lambda$ to $+\lambda$ for $\alpha = 1$. The upper panel corresponds to $\gamma = 1$ where there are periodic occurrences of DPTs when $\lambda = 2$ while DPTs get rounded off when $\lambda = 2.5$. The lower panel corresponds to the anisotropic situation with $\gamma = 0.5$ where the presence of DPTs are wiped out when $\lambda = 1.8 (> (1 + \gamma))$ or $\lambda = 0.4 (< (1 - \gamma))$ while these are prominently present when for $\lambda = 1.5 (= (1 + \gamma))$ and $\lambda = 0.5 (= (1 - \gamma))$ as discussed in the text. It should also be noted that the position of the maxima (or non-analyticities) depend on the magnitude λ .

provides a unique example of a situation where DPTs could be absent even when an integrable model is slowly ramped across a QCP.

Concerning the sudden quenches we find that even in the isotropic case the presence of DPTs is not guaranteed; neither the situation is like a sudden quench through a single QCP as in the case of slow quenches. Rather both

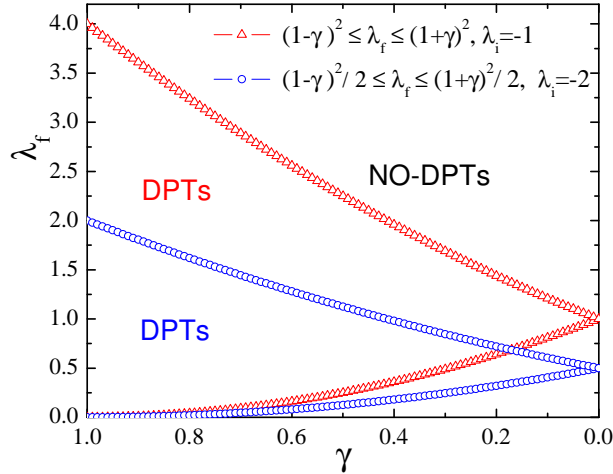


FIG. 5: (Color online) The phase diagram in the $\lambda_f - \gamma$ plane showing the regions where DPTs will occur following sudden quenches with $\lambda_i = -1$ (red triangles) and $= -2$ (blue circles) to a final value λ_f . In both the cases, DPTs occur when λ_f lies within the range $(1 - \gamma)^2/|\lambda_i|$ and $(1 + \gamma)^2/|\lambda_i|$ as elaborated in the text. In the isotropic situation ($\gamma = 1$), the condition gets simplified to $0 \leq \lambda_f \leq 4/|\lambda_i|$.

in the isotropic and anisotropic cases, one finds restrictions on the values of λ_i and λ_f depending on the parameter γ determined from Eqs. (12) and (13). It is never important whether the spin chain is quenched across the QCP in the process of quenching; however, it should necessarily be swept through $\lambda = 0$, i.e., either λ_i or λ_f should be negative for DPTs to appear. We have illustrated these different scenarios in Fig. 5. This is remarkable that DPTs can be made to appear (or disappear) in the same model by tuning either the anisotropy term γ or the inverse quench rate τ for slow quenches, and λ_i and λ_f for sudden quenches such that the system must be driven across $\lambda = 0$.

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