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Quantifying Risk due to Capacity Uncertainty on Urban Road Networks

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Abstract

The urban transportation network is susceptible to degradation in its performance because of various measures. It would be of interest to a planner if this risk can be quantified. In this paper, we present a methodology to do so by estimating the n most probable states of a network. A particular network state is a combination of capacity levels on all links and is defined by defining the different capacity levels at which all the links are operating at. The probability of occurrence of that state would be the product of probabilities of occurrence of capacities of all links. Suppose there are m links and n capacity levels. Each link can function in any of the n capacity levels. So the maximum number of states enumerated would be n^m . A most probable state (MPS) is that state which has highest probability to function on a network. For each state consequence were found out. Consequence is the extent of variation from the minimum system travel time. Thus a graph is plotted between the consequence and the probability of its occurrence for all the states of the network. Risk associated to the network is defined by the probability for an event of negative impact to occur and the extent of the resulting consequence once this event has taken place. The routing assumed was static user equilibrium.

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1. Introduction

Roadways form the links of urban transportation networks. These links are susceptible to disruption in their service levels due to many physical reasons ranging from infrastructural repairs, natural events, road incidents, illegal parking (roadside infiltration), and/or their combinations.

On such fragile networks, it is important for transportation planners and analysts to quantify the risk associated with the fragility. Risk associated with any negative event depends on the probability of the event to occur and the extent of the consequence of the event (Berdica, 2012). In the problem dealt in this paper, the negative impact is the disruption of capacity on any link(s) and the consequence is the increase in travel time due to the disruption on that link.

In an urban transportation network consisting of multiple links and nodes, it is possible that the realised capacity on link would be lower than the designed. The capacities on links are stochastic. Chances are high for the capacity to follow some pattern of frequencies but the deterministic state outcomes are hard to predict. The estimation of capacity values at a specific site can be obtained by the traffic observation from that site using direct-empirical studies (Minderhound, Botma and Bovy, 1997). The impact of the disruption on the flows and densities of traffic networks have been studied by Zhang, Garoni and Gier (2013).

A modelling of network performance is done by Lo and Tung, (2003) in the face of degradation of link capacities. They formulated the approach of *probabilistic* user equilibrium considering that the commuters get used to the travel time variability based on the past experience. So this would affect the route choice model. A probabilistic approach was adopted to model the factors that degrade the capacities of roadways by Chen et al., (1999). He introduced about measuring capacity reliability which is the probability that a certain traffic demand can be accommodated at an acceptable level of service.

Lo et al. (2006) modelled risk aversion behaviour of travellers on networks susceptible to uncertainty in capacity-realizations, assuming standard distributions with known values of parameters on capacity degradation. Chen et al. (2002)—while assessing capacity reliability of a road network—assumed roadway capacities as random variables that follow probability distribution with specified correlation among them. Recently, Ng et al. (2011) introduced a distribution-free methodology in which the model required N-moments of travel time data. All these instances involved explicit or implicit assumption of familiarity on capacity realization pattern. Other approaches involved to analyze network with stochastic capacities are Method of successive averages (MSA) by Meng et al., (2001), Within Budget Time Reliability (WBTR) by Lo, Luo and Siu, (2006) and by Siu and Lo (2008).

A network state is defined by defining the capacity levels on all the links of the network. A combination of different capacity levels would make multiple states. The network states are enumerated using algorithm ORDER-M (Chiou, and Li, 1986).

Performance measure in a road network can be taken to be the system travel time, the total cost of travel associated with the network, the distance travelled from origin to destination, safety, sustainability and environmental quality, accessibility, reliability, etc. Our prime consideration for the performance measure would be system travel time.

Traffic disruptions cause bottlenecks, which reduce the network capacity, and usually result in traffic jam. The performance of a network largely depends on the capacity level at which the links are operating. If the links are operating at the designed capacity, the network is highly reliable.

The traffic assignment model used is static user equilibrium. It is assumed that the network link capacity stochastic whereas the origin-destination (OD) demand is deterministic. This study assumes that the link capacity distributions for different links are independent.

The model proposed in this paper presents a methodology to calculate the consequence of the disruption of the capacity on any link. This can be used to compare the robustness of different networks. The study mainly attempts to analyze the impact of disruption of capacity in an urban transportation network. The study draws its motivation from the fact that these are some of the most concerning issues to the traffic planner. This study would help the traffic planner to design the multilane urban transport network considering the vulnerability of the disruption of capacities on links. In Section 2 the methodology is presented. Section 3 gives the solution to the methodology adopted and presents example results on sample networks. Section 4 outlines discussions from these results and section 5 gives the conclusions and future research scope.

2. Methodology

The methodology is sequenced as follows: (a) we find the probability of occurrence of capacity level, (b) we then enumerate the most probable state given the probability of each capacity level on all links, (c) we assign traffic following user-equilibrium assignment and determine the consequence of each state (d) we thus compare the risk of different disruption states. The same is presented in figure 1 in the form of flow-chart.

Due to disruption in a link, the operable capacity degrades from the designed. The extent of disruption can vary with the extreme state being inoperable (operable capacity = 0 % of designed capacity) and the best state being no disruption at all (operable capacity = 100% of designed capacity) on the link.

We consider three scenarios: in scenario I to III, the chances of disruption levels vary as low, medium, and high which signifies that the probability of occurrence of higher capacity levels is more in scenario I than in scenario III. We define different levels of capacity on a link. For instance, the operable capacity with three levels would be: Level 1: 0-33% of designed capacity, level 2: 33-36% of designed capacity, level 3: 36-100% of designed capacity. Accuracy may be improved by calibrating the number of possible levels from field observations. n capacity levels would result in the i^{th} operable capacity range being $((i-1)*\frac{100}{n}$ to $(i)*\frac{100}{n}$) % of C_d where C_d is the designed capacity of the link.

Upon dividing the whole range of capacity into different levels, we calculate the probability of occurrence of each capacity level on all the links. This is done by assuming the operable capacity on a link to follow a normal distribution curve (Minderhound, Botma and Bovy, 1997). The area enclosed under the curve between two vertical lines give the probability of occurrence of that level. In scenario I, the probability distribution curve is shifted towards right whereas in scenario III, it is shifted towards left.

Any particular state would be defined by defining the capacity levels at which all the links are operating at. The probability of occurrence of that state would be the product of probabilities of occurrence of capacities at all links. Suppose there are m links and n capacity levels. The maximum number of states possible would be n^m . But we need to enumerate states only until some significant probability value as the probability for later states would be too low. To find the most probable state, various works have been done by Li and Silvester (1984), Gomes, Craveirinha and Martins (2002) and Yang and Kubat (1989). The algorithm derives its use in many other networks including communication networks. The one that works for our case is Order-M algorithm (Chiou and Li, 1986).

A program calculates the M most probable state given the probability values in each link. A test network with assumed capacity distribution is considered to illustrate the computation of probable states.

The traffic routing done is static user equilibrium which follows Wardrop's first principle, (1952). The flow on links is obtained based on the operable capacity of the most probable state of a particular scenario. The same flow is assigned to all other enumerated states of that scenario to find the travel times on all the links of each state. This is done since for a particular scenario, users would experience traffic flows, for any state, same as the traffic flows for the most probable state.

We use the Bureau of Public Roads (BPR) function (Traffic Assignment Manual, 1964) to calculate the travel time given the base travel time, flow and operable capacity on all links.

$$TT = FFT (1 + \alpha (\frac{v}{c})^\beta) \quad (1)$$

where,

TT = predicted mean travel time; FFT = free-flow travel time; v = volume; c = capacity; α , β = parameters ($\alpha = 0.15$, $\beta = 4$). The travel time of all the links of the network are multiplied by the flow on those links and summed up to obtain the total system travel time. The minimum system travel time is obtained by assigning flows in the network considering the design-capacity on all the links i.e., links are operating without any disruption at all.

Risk is a combination of probability and consequence, the probability of the occurrence of system travel time in any range and the consequence is the extent of variation of that range from the minimum system travel time of the network. The consequence of each state is calculated by deducting this minimum STT from the STT of each state. A bound of consequence and the probability of their occurrence is obtained.

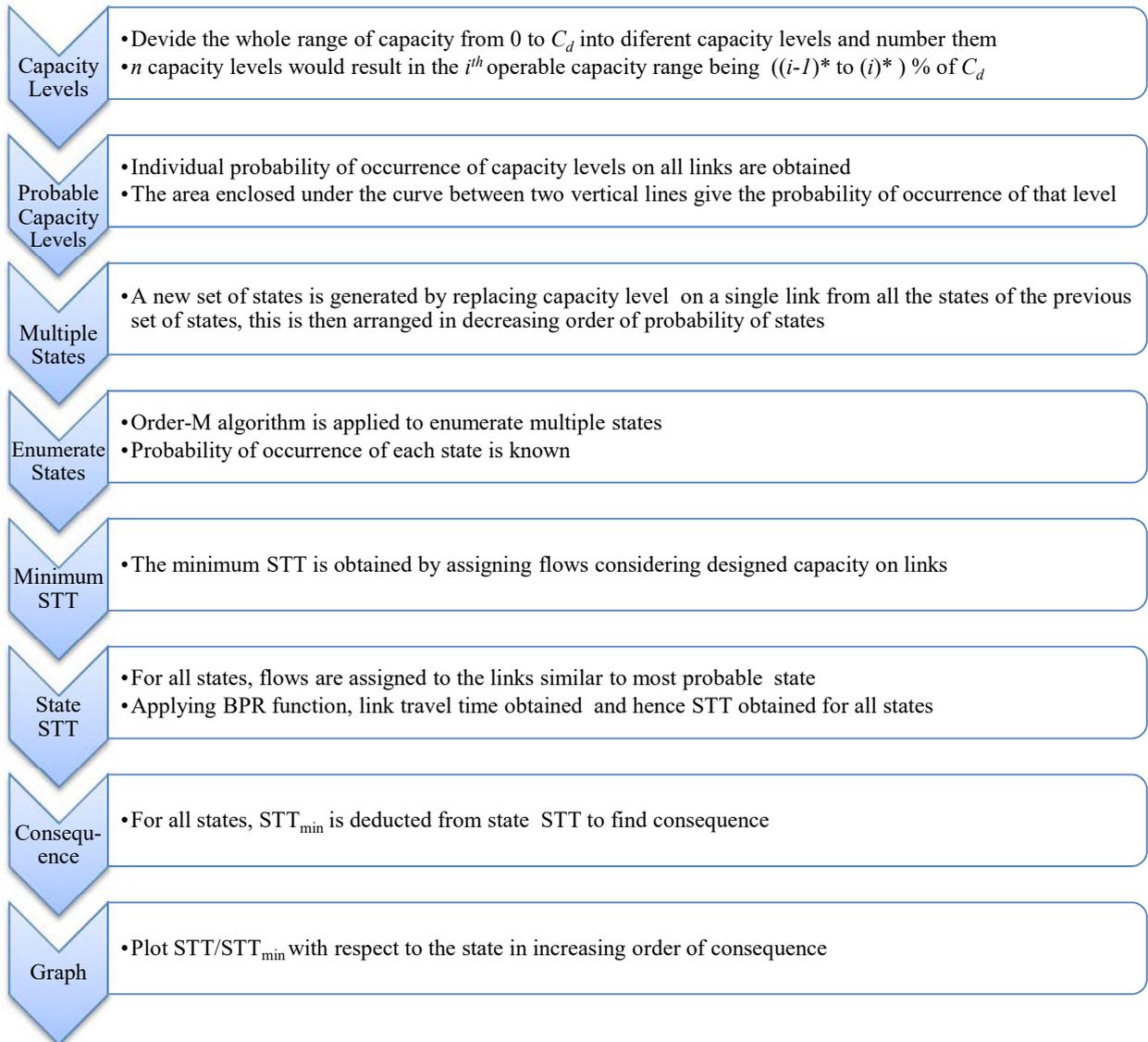


Figure 1: Flow-chart of methodology

3. Analysis on Network

We perform a sample analysis on Braess Network as shown in Figure 2. We assume three kinds of scenario for the network. Scenario I represents that the links are least disruptive whereas scenario III represents that the links are most disruptive.

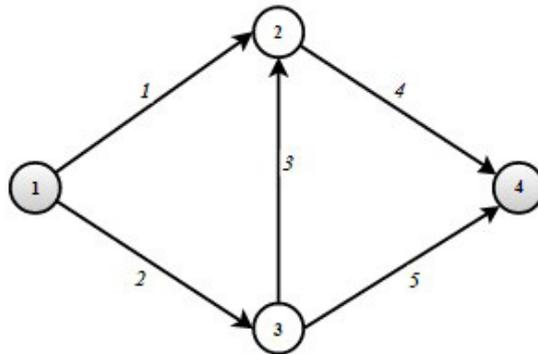


Figure 2: Braess Network consisting of 5 links and 4 nodes

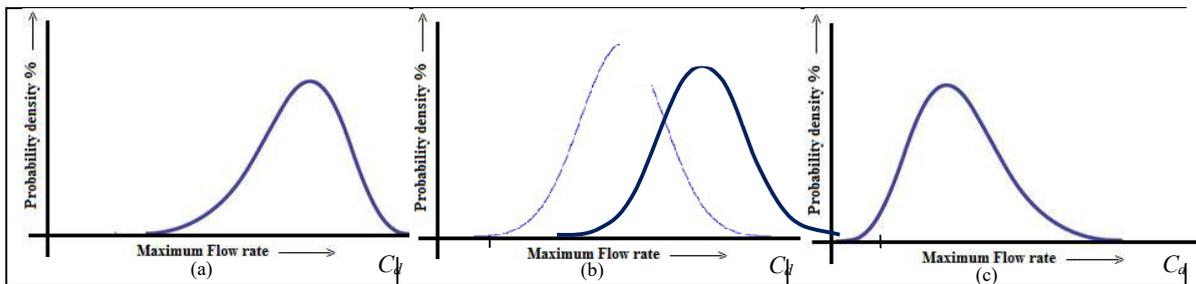


Figure 3: Capacity as a stochastic variable

Figure 3(a) shows the plot of probability density function for capacity variations on link for scenario I, where the network is least disruptive, hence the distribution of capacity is shifted towards the right. Figure 3(b) shows the plot for scenario II and figure 3(c) for scenario III, where the network is most disruptive, hence the distribution of capacity is shifted towards the left. The mean and standard deviations of the operable capacities on all five links for all the three scenarios of the network are assumed such that they follow normal distribution function.

In each scenario, four levels of capacity are defined on all links. Different levels of capacity on links can be defined for different analysis. Capacity level 1 is for the network being most disruptive, i.e., the operable capacity lies in 0-25% of the designed capacity and capacity level 4 is for the network being least disruptive, i.e., the operable capacity lies in 75-100% of the designed capacity. The probability of occurrence of capacity levels are found on all the five links for all the three scenarios.

In scenario I, where we assumed least traffic disruption, significant probabilities are for higher capacity levels whereas for scenario III, significant probability values are for lower capacity levels since we assumed high traffic disruption for this scenario.

Applying the MPS algorithm, several states are enumerated such that the cumulative probability comes to be a significant number. In the Braess network, the 100th state has a cumulative probability of 0.99. A sample of 18 states in between the state number 1 to 100 for all the three scenarios is taken. The operable capacity level of each link for all the states is found along with the cumulative probability of their occurrence. Suppose, for scenario I, the operable capacity level on links for the most probable state is {3 3 3 4 3}, this suggests that on links 1, 2, 3 and 5 capacity level is 3 and on link 4, the capacity level is 4.

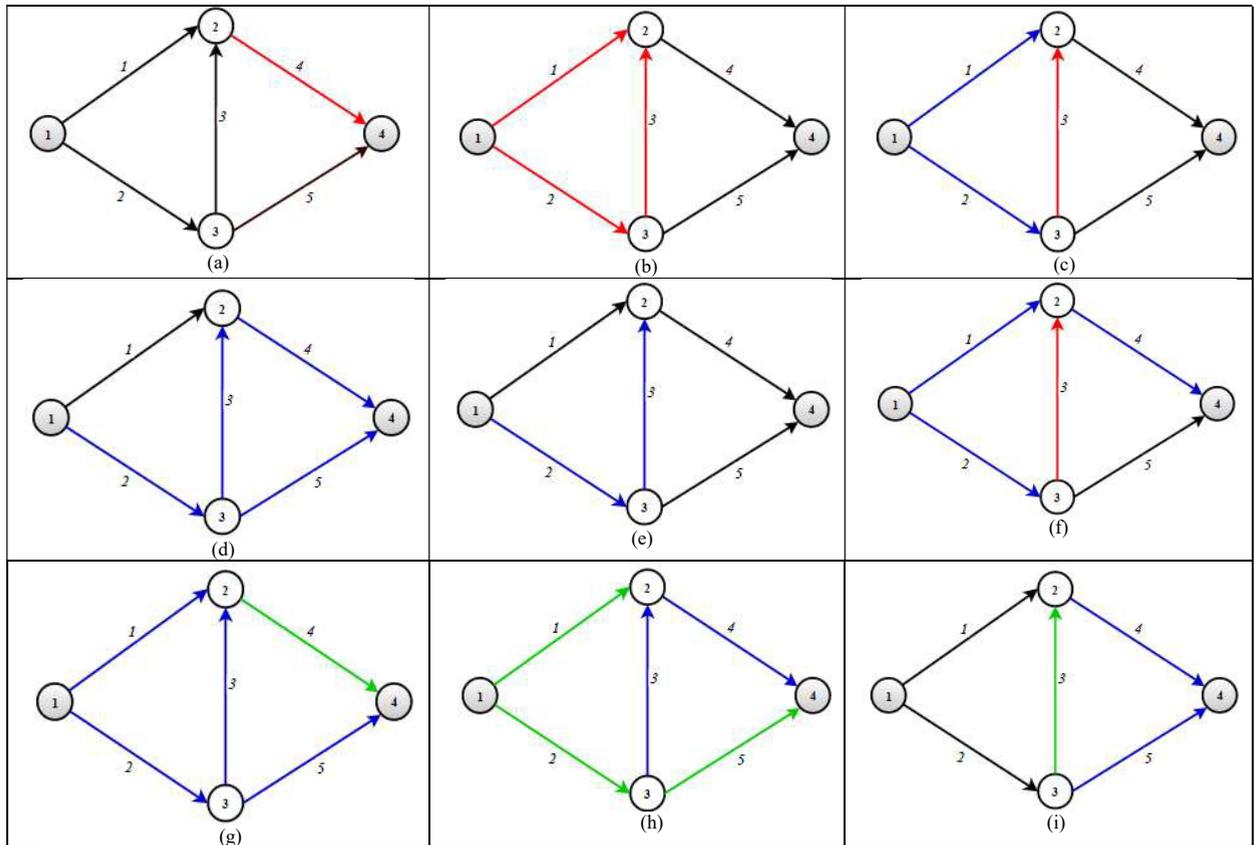


Figure 4: Operable capacities for different states. Green: Capacity level : 1 (1.5); Blue: Capacity level : 2 (4.5); Black: capacity level : 3 (7.5); Red : Capacity level : 4 (10.5)

Figure 4 shows the operable capacities or capacity levels on links for different states. Figure 4 (a), (b), (c) resembles scenario I for Most probable state, 30th probable state and 100th probable state respectively. Likewise figure 4 (d), (e), (f) resembles scenario II and figure 4 (g), (h), (i) resembles scenario III. The corresponding values of operable capacities for the different network states are obtained.

Traffic is assigned to the most probable states using static user equilibrium model and the link travel times are obtained. The same flows are assigned to the other states of the respective scenarios and the link travel times are obtained using BPR function (Traffic Assignment Manual, 1964). The system travel time is obtained for all states corresponding to their probabilities of occurrence.

The minimum system travel time is computed by considering that all the links are operating at the designed capacity levels i.e., at 100%. The minimum STT comes to be 60.0354. The consequence of each state is calculated by deducting the variation in the STT of each state and the min STT of the network. Now we have the consequence of each state along with the probability of their occurrence. Further a factor STT / STT_{\min} is calculated to compare different scenarios.

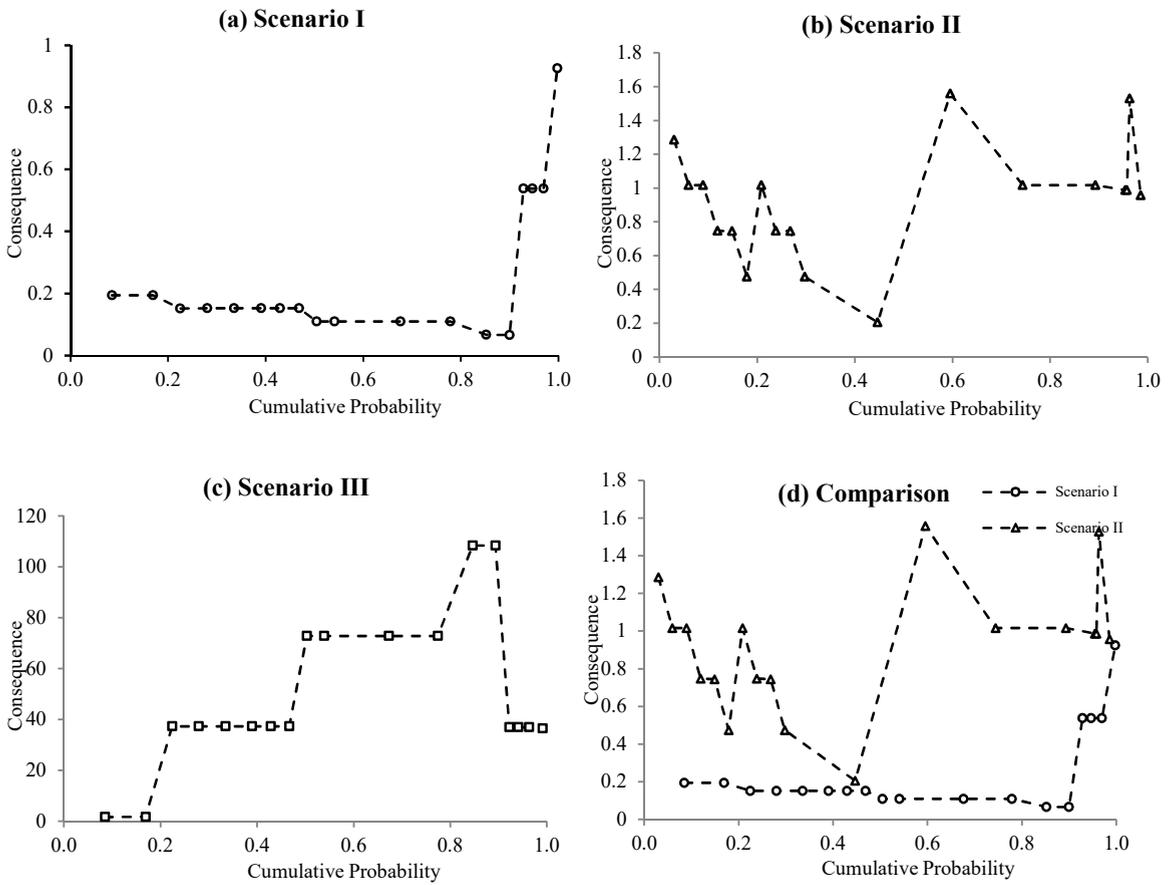


Figure 5: Graphs showing plot of cumulative probability vs consequence for the three scenarios.

A plot with cumulative probability on x-axis and consequence on y-axis for all the three scenarios is plotted. Figure 5 (a) (b) (c) represents the variation of consequence with the cumulative probabilities for scenario I, II and III respectively. The consequence of scenario I is lesser than scenario II which again is lesser than scenario III. This is in consistent with our assumption that the chances of disruption of capacity on links in scenario I is the least. So the operable capacities on links are close to the designed capacity due to which the system travel time of the states does not differ much from the minimum travel time of the network.

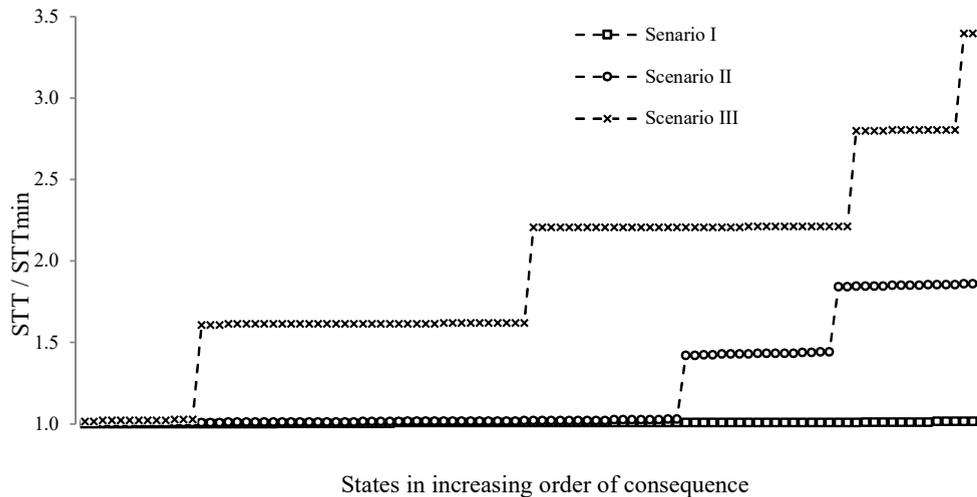


Figure 6: A plot of states in increasing order of consequence with respect to the STT factor

For scenario III, it is observed that the STT shoots up to more than 3 times of the minimum STT. The link disruption in network can lead to a catastrophic increase in the system travel time. Figure 6 shows the plot for all the three scenarios. For scenario I, it was assumed that the disruption level was the least, so after calculating the 100 most probable states (up to a cumulative probability of 0.99) and computing their STT, it is observed that all the probable states do not vary from the minimum system travel time much. In scenario II, some probable states might have their STT to be about two times of the minimum STT. Whereas in scenario III, the links were exposed to maximum disruption in their capacity levels, so there are states which have STT even more than three times of the minimum STT. This might happen in urban transportation network where the links are most susceptible to disruptions. One can expect traffic breakdown with the STT escalating to a very high value. A traffic planner needs to consider such networks for retro-fitting or capacity expansion.

4. Discussions

The consequence varies with the extent of disruption of the link. If the links have maximum disruption, the network goes to a critical state where the STT would shoot up to a very large number in comparison to the minimum STT.

The routing assumed was static user equilibrium. However a question regarding the transition time required remains. Static User Equilibrium requires a transition time in which users would equilibrate among themselves. However the capacity disruption that are being considered in this study need not hold for a longer period. Which routing scheme would better represent the resultants of ad-hoc capacity levels on a networks' link, need to be relooked. Probabilistic User Equilibrium (PUE) by Lo and Tung, (2003) – in which the users try to minimize the difference of expected path travel times – can be an alternate routing scheme, which may hold under such conditions.

5. Conclusion

The capacity of Indian urban roads is uncertain due to three reasons: (a) dynamic nature of parameters that influence capacity, (b) lack of a versatile unit that homogenize vehicular heterogeneity and (c) the inherent vagueness in modelling/observing the capacity. The capacity uncertainty leads to an uncertainty in network's operational cost.

This paper illustrates the methodology to quantify risk on urban transport network. The effect on performance of a network – based on system travel time – due to disruption of any component is a vital area to study. This methodology can be used to compare different networks' performance and can justify whether the network needs retro-fitting or capacity expansion.

A common assumption in the current methodology dealing with stochastic transportation systems is that the exact probability distributions of the capacity on links are known explicitly. However, the probability distributions may be unavailable (inaccurate) in reality because we may have no (insufficient) data to calibrate the distribution. In the study, the link capacity disruptions are considered to be independent to each other. This would change in the real situation since the disruption of capacity of one link would affect another.

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