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Stochastic $\mathscr{H}_\infty$ control of state-dependent jump linear systems with state-dependent noise

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Abstract: State-dependent jump linear systems (SJLSs) are a set of linear systems whose switching is determined by a finite state random process with state-dependent transition rates. In this study, a SJLS is considered with state multiplicative noise and stochastic perturbations. In particular, the jump process is regarded to have dissimilar transition rates among sets of state evolution space. The aim of this study is to consider stochastic $\mathscr{H}_\infty$ control of such systems using state-feedback control input. Sufficient conditions for stochastic $\mathscr{H}_\infty$ are obtained by solving linear matrix inequalities, which are validated by a simulation example.

1 Introduction

Stochastic switching systems effectively model dynamics with abrupt changes in their working modes. Such systems have widespread applications, for instance, in fault tolerant control [1, 2], networked systems [3], manufacturing systems [4], economics [5]. In switching systems literature, Markov jump linear systems (MJLSs) are widely studied where set of systems are linear and switching follows a homogeneous Markov process with finite states. Various applications of MJLSs can be found in [4, 6, 7] for instance. Also, for [8, 9] and references therein deal with several results related to control design and stability analysis of MJLSs. In MJLSs, the switching process is homogeneous Markovian, which is rather a restriction to apply it to more general scenarios.

As a way to be more general, a class of switching systems considered in this article are called state-dependent jump linear systems (SJLSs), where the set of systems is linear and switching process is state-dependent. Such SJLS modelling stems from the following scenarios. State-dependent failure rate of components is considered in [10], in submarine engines, random failures are modelled as state-dependent Markov process [11], also state-dependent switching [12] is employed in modelling of financial time series. Several other examples or scenarios of state-dependent regime switching can be observed in other applications.

Available works related to stability analysis and control design of SJLSs have been scanty, which are reviewed here. Uniqueness and ergodicity of a non-linear system with diffusion and state-dependent switching is addressed in [13]. For flexible manufacturing systems with state-dependent failures, a control design via dynamic programming is addressed in [14]. For a jump system subject to diffusions with state-dependent transitions and dual time scales, an optimal control is addressed in [15]. For a case of switching rate of the underlying jump process depending both on system state and input, an optimal control policy is addressed in [16]. For SJLSs, a model predictive control problem is considered in [7], while stability and robust stabilisation for SJLSs are addressed in [17–19].

On the other hand, systems affected by multiplicative noise have attracted a lot of attention in widespread applications including altitude estimation, guidance motivated tracking filter, terrain following, adaptive motion control to name a few, see, for instance [20–22] and reference therein. A well established $\mathscr{H}_\infty$ analysis of linear systems affected by state multiplicative noise is addressed in [20, 23] for instance. For MJLSs, the same has been investigated by [9] for instance. However, $\mathscr{H}_\infty$ analysis to SJLSs with state multiplicative noise is yet to be addressed. Compared to the existing works, this paper focuses on the consideration of state multiplicative noise for SJLSs, which has not been addressed so far.

In particular, the state-dependent switching rates are considered as follows: the state evolution space is divided as finite sets and the switching rates variation depends on each such set. It is a legitimate assumption to make, since at any time the state variable traverses one of these sets, where transitions rates are considered to be dissimilar in each set. With the given assumptions, a $\mathscr{H}_\infty$ control synthesis is addressed via Dynkin's formula and given in terms of linear matrix inequalities (LMIs).

The paper is organised as follows. A mathematical description of SJLS and $\mathscr{H}_\infty$ control problem is provided in Section 2. Precursory results are provided in Section 3 and further the major results are given in Section 4. A simulation example is furnished in Section 5, while the conclusions are provided in Section 6.

Notation: $\mathbb{R}_+$ stands for the positive real line. For a random vector or scalar x, $\mathbb{E}[x]$ denotes its expectation. $A^\top$ indicates the transpose of a matrix A, $\lambda_{\min}(A)$ denotes the least eigenvalue of A. $\mathbb{I}_n$ designates the identity matrix of size $n \times n$ and $\mathbb{I}$ denotes an identity matrix of suitable size. For a matrix $\mathscr{P}$, which is real and symmetric, $\mathscr{P} \succ 0$ ($\mathscr{P} \prec 0$) denotes that $\mathscr{P}$ is positive definite (negative definite), respectively. In a matrix, $\star$ denotes symmetric terms. $\mathrm{diag}\{P_1, P_2, \ldots, P_n\}$ denotes diagonal matrix formed by $P_1, P_2, \ldots, P_n$. ϕ represents the null set. For two scalars x and y, $x \wedge y$ represents the minimum of x and y.

2 Problem formulation

In this section, dynamics of SJLS are provided.

In a probability space $(\Omega, \mathscr{F}, \Pr)$, consider a SJLS with a state-dependent noise and stochastic perturbations:

$$\begin{aligned} \mathrm{d}x(t) = {} & A_{\theta(t)}x(t)\,\mathrm{d}t + \tilde{A}_{\theta(t)}x(t)\,\mathrm{d}w_1(t) + \tilde{B}_{\theta(t)}v(t)\,\mathrm{d}w_2(t) \\ & + B_{\theta(t)}v(t)\,\mathrm{d}t + E_{\theta(t)}u(t)\,\mathrm{d}t \end{aligned} \tag{1a}$$

$$z(t) = C_{\theta(t)}x(t) + D_{\theta(t)}v(t) + F_{\theta(t)}u(t)\,. \tag{1b}$$

Such models without switching process are often result from linearisation, see, for instance, a tracking problem in [24] and robust control design in [23]. Here the state vector $x(t) \in \mathbb{R}^{n_x}$, the control input $u(t) \in \mathbb{R}^{n_u}$, the performance output $z(t) \in \mathbb{R}^{n_z}$, initial state $x(0) = x_0$, stochastic disturbance $v(t) \in \mathbb{R}^{n_v}$, $w_1(t)$ and $w_2(t)$ are real scalar Wiener processes with zero mean where $\mathbb{E}[(w_l(t) - w_l(s))(w_m(t) - w_m(s))] = q_{lm}(t - s)$, $l, m = 1, 2$, $t, s \in \mathbb{R}_+, t >$, and the system matrices $A_{\theta(t)} \in \mathbb{R}^{n_x \times n_x}$, $\tilde{A}_{\theta(t)} \in \mathbb{R}^{n_x \times n_x}$, $\tilde{B}_{\theta(t)} \in \mathbb{R}^{n_x \times n_v}$, $B_{\theta(t)} \in \mathbb{R}^{n_x \times n_v}$, $E_{\theta(t)} \in \mathbb{R}^{n_x \times n_u}$, $C_{\theta(t)} \in \mathbb{R}^{n_z \times n_x}$, $D_{\theta(t)} \in \mathbb{R}^{n_z \times n_v}$ and $F_{\theta(t)} \in \mathbb{R}^{n_z \times n_u}$ dependent on $\theta(t)$, which are considered to be known. Let a mode process of the system $\theta(t)$ be $\{\theta(t), t \geq 0\} \in S := \{1, 2, \ldots, n_S\}$, and the switching between different modes of (1) dependent on the system state as

$$\Pr\{\theta(t+\epsilon) = j | \theta(t) = i, x(t)\} = \begin{cases} \mu_{ij}^1 \epsilon + o(\epsilon), & \text{if } x(t) \in \mathscr{C}_1, \\ \vdots & \\ \mu_{ij}^K \epsilon + o(\epsilon), & \text{if } x(t) \in \mathscr{C}_{n_K}, \end{cases} \tag{2}$$

where $\epsilon > 0$, $\lim_{\epsilon \to 0} (o(\epsilon)/\epsilon) = 0$, $\mathscr{C}_1, \mathscr{C}_2, \ldots, \mathscr{C}_{n_K} \subseteq \mathbb{R}^{n_x}$ are non-empty Borel sets, where each of them is a connected set that span $\mathbb{R}^{n_x}$ and disjoint, i.e. $\cup_{\iota=1}^{n_K} \mathscr{C}_\iota = \mathbb{R}^{n_x}$ and $\mathscr{C}_l \cap \mathscr{C}_m = \phi$ for any $l, m \in \mathscr{K} \triangleq \{1, 2, \ldots, n_K\}$, $l \neq m$. For $m \in \mathscr{K}$, $i, j \in S$, μ_{ij}^m is the switching rate of $\theta(t)$ from i to j where $\mu_{ij}^m \geq 0$ for every $i \neq j$ with $\mu_{ii}^m = -\sum_{j=1, j \neq i}^{n_S} \mu_{ij}^m$.

First, examine system (1) for the case of no control input

$$\begin{aligned} \mathrm{d}x(t) = A_{\theta(t)} x(t)\, \mathrm{d}t + \tilde{A}_{\theta(t)} x(t)\, \mathrm{d}w_1(t) + \tilde{B}_{\theta(t)} v(t)\, \mathrm{d}w_2(t) \\ + B_{\theta(t)} v(t)\, \mathrm{d}t \end{aligned} \tag{3a}$$

$$z(t) = C_{\theta(t)} x(t) + D_{\theta(t)} v(t). \tag{3b}$$

Remark 1: In SJLS (3), the perturbation process $v(t) \in \mathbb{R}^{n_v}$ is considered as a stochastic noise in this paper in-line with model of [23] for linear systems. The system dynamics (3) consists of multiplicative state noise terms and the stochastic disturbance terms that may be viewed as system matrix perturbations involving white noise as

$$\mathrm{d}x(t) = (A_{\theta(t)} + \tilde{A}_{\theta(t)} \dot{w}_1(t)) x(t)\, \mathrm{d}t + (B_{\theta(t)} + \tilde{B}_{\theta(t)} \dot{w}_2(t)) v(t)\, \mathrm{d}t.$$

Define $\mathscr{L}_2^s(0, T)$ as a space of adapted processes $y(.) = (y(t))_{t \in [0, T]}$ adapted to σ-algebras $(\bar{\mathscr{F}}_t)_{t \in [0, T]}$, where $\bar{\mathscr{F}}_t \subset \mathscr{F}$ with $t \in \mathbb{R}_+$ satisfying

$$\left\| y(.) \right\|_{\mathscr{L}_2^s}^2 \triangleq \mathbb{E}\left(\int_0^T \| y(t) \|^2 \, \mathrm{d}t \right) < \infty.$$

Definition 1: The system (3) is called internally stable if, for $v(t) = 0$, for any $\theta(0) \in S$ and $x_0 \in \mathbb{R}^{n_x}$,

$$\mathbb{E}\left[\int_0^\infty \| x(t) \|^2 \, \mathrm{d}t \right] < \infty.$$

Definition 2: The system (3) is called externally stable if, for every $v(.) \in \mathscr{L}_2^s(0, \infty)$, for any $\theta(0) \in S$ and zero initial state condition, $\exists$ a real scalar $\gamma \geq 0$ satisfying

$$\left\| z(.) \right\|_{\mathscr{L}_2^s}^2 \leq \gamma \left\| v(.) \right\|_{\mathscr{L}_2^s}^2. \tag{4}$$

The objective of this paper is to find a minimum γ such that (4) is satisfied while being internally stable, which we call *stochastic* $\mathscr{H}_\infty$ *problem*. In relation to (4), let the perturbation operator $\| \mathbb{L} \|$ be

$$\| \mathbb{L} \| = \sup_{v \in \mathscr{L}_2^s, v \neq 0} \frac{\| z \|_{\mathscr{L}_2^s}}{\| v \|_{\mathscr{L}_2^s}},$$

whose norm is the minimum $\gamma \geq 0$ such that (4) is satisfied, where z and v are given according to Definition 2.

Consider the following integral:

$$J(T) = \int_0^T \mathbb{E}\left[\left\| z(t) \right\|^2 - \gamma^2 \| v(t) \|^2 \right] \mathrm{d}t, \tag{5}$$

for $T \to \infty$, where it is shown in later sections that minimising (5) will lead to the solution of stochastic $\mathscr{H}_\infty$ problem.

3 Preliminaries

In this section, to tackle the state-dependent transitions (2) in the current setting, the descriptions of SJLS (3) and mode $\theta(t)$ (2) are slightly altered, which leads to an equivalent model of (3).

Consider a finite state process $\zeta_t \in \mathscr{K}$ denoting the partition the state belongs at time t as

$$\zeta_t = \begin{cases} 1, & \text{if } x(t) \in \mathscr{C}_1, \\ \vdots & \\ n_K, & \text{if } x(t) \in \mathscr{C}_{n_K}. \end{cases}$$

Let $r(\zeta_t, t) \in S$ (equivalent to $\theta(t)$) be a finite state random process with state-dependent switching whose switchings depend on ζ_t for $i \neq j$,

$$\Pr\{r(\zeta_{t+\epsilon}, t+\epsilon) = j | r(\zeta_t, t) = i, \zeta_t\} = \begin{cases} \mu_{ij}^1 \epsilon + o(\epsilon), & \text{if } \zeta_t = 1, \\ \vdots & \\ \mu_{ij}^{n_K} \epsilon + o(\epsilon), & \text{if } \zeta_t = n_K, \end{cases} \tag{6}$$

where μ_{ij}^m, for $m \in \mathscr{K}$, are described in (2). Thus, SJLS (3) is rewritten as

$$\begin{aligned} \mathrm{d}x(t) = A_{r(\zeta_t, t)} x(t)\, \mathrm{d}t + \tilde{A}_{r(\zeta_t, t)} x(t)\, \mathrm{d}w_1(t) \\ + \tilde{B}_{r(\zeta_t, t)} v(t)\, \mathrm{d}w_2(t) + B_{r(\zeta_t, t)} v(t)\, \mathrm{d}t \end{aligned} \tag{7a}$$

$$z(t) = C_{r(\zeta_t, t)} x(t) + D_{r(\zeta_t, t)} v(t). \tag{7b}$$

The analysis of system (7) with mode process (6) tantamounts to analysing system (1) with mode process (2). As can be seen in later sections, this equivalence facilitates the derivation of main results in a non-clutter manner.

Remark 2: From (6), observe that for $r(\zeta_t, t) = i \in S$, $r(\zeta_{t+\epsilon}, t+\epsilon)$ relies on the state variable $x(t)$ for any $\epsilon > 0$, which further relies on $r(\zeta_s, s)$, $s < t$ from (7). Thus the process $r(\zeta_t, t)$ is not Markovian.

For $l \in \mathscr{K}$ and $t_2 \geq t_1 \geq 0$, $\Psi_l(t_1, t_2)$ describes a flow of system (7) on the interval $[t_1, t_2]$, for the switching rate of $r(\zeta_t, t)$ being μ_{ij}^l when $\zeta_t = l$, for $i \neq j \in S$. Using the flows of system (7), first exit times $\tau_0, \tau_1, \ldots$ are defined as follows.

For $m = 0, 1, 2, \ldots$, given τ_{m-1}, $i_{m-1} \in \mathscr{K}$, let $x(\tau_{m-1}) \in \mathscr{C}_{i_m}$, where $i_m \neq i_{m-1}$, $i_m \in \mathscr{K}$. Let τ_m be the first exit time of $x(t)$ from set $\mathscr{C}_{i_m}$ after τ_{m-1} as

$$\begin{aligned} \tau_m = \inf\{t \geq \tau_{m-1} \colon \Psi_{i_m}(t, \tau_{m-1}) \Psi_{i_{m-1}}(\tau_{m-1}, \tau_{m-2}) \\ \cdots \Psi_{i_0}(\tau_0, 0) x(0) \notin \mathscr{C}_{i_m}\}, \end{aligned} \tag{8}$$

where

$$\tau_0 = \inf\{t \geq 0 \colon \Psi_{i_0}(t, 0) x(0) \notin \mathscr{C}_{i_0}\}.$$

Remark 3: From (8), $\{\zeta_t, t \geq 0\}$ can be given in an alternative form as

$$\zeta_t = \begin{cases} i_0, & \text{if } t \in [0, \tau_0), \\ \vdots & \\ i_m, & \text{if } t \in [\tau_{m-1}, \tau_m), \\ \vdots & \end{cases}$$

where $\{i_0, i_1, \ldots, i_m, \ldots\} \in \mathcal{K}$. Also k^* represents the number of switchings attained by ζ_t.

Let $\mathcal{F}_t$ be the natural filtration of $(x(t), r(\zeta_t, t), \zeta_t)$, a solution of (6) and (7), over $[0, t]$. Then it can be shown by applying the steps of Lemma 1 of [18] that $(x(t), r(\zeta_t, t), \zeta_t)$ is a joint stochastic process adapted to $\mathcal{F}_t$. Also, continuing the treatment of the first exit times, it can be observed that the first exit times $\tau_0, \tau_1, \tau_2, \ldots$ are $\mathcal{F}_t$-stopping times since the events $\{\tau_i \leq t\}$ for $i = 0, 1, 2, \ldots$ are $\mathcal{F}_t$-measurable.

Definition 3: Let a joint stochastic process $\eta(t) \triangleq (x(t), r(\zeta_t, t), \zeta_t)$, a solution of (6) and (7), be a joint Markov process and τ_0, τ_1, $\tau_2, \ldots$ be the stopping times, then for an admissible Lyapunov function $V(\eta(t))$, by the Dynkin's formula [25, 26], we can write

$$\begin{aligned} &\mathbb{E}[V(\eta(t))|\eta(0)] - V(\eta(0)) \\ &= \sum_{k=0}^{k^*} \mathbb{E}\left[\int_{t \wedge \tau_{k-1}}^{t \wedge \tau_k} \mathscr{A}V(\eta(s))\,\mathrm{d}s \middle| \eta(s)\right], \end{aligned} \tag{9}$$

where $k = 0, 1, \ldots, k^*$, $k^* \in [0, \infty]$, $\tau_{-1} = 0$ and $\tau_{k^*} \leq \infty$, with domain $[0, \infty) \times \mathbb{R}^n \times S \times \mathcal{K}$ and

$$\mathscr{A}V(\eta(t)) = \lim_{\mathrm{d}t \to 0} \frac{1}{\mathrm{d}t}\left\{\mathbb{E}\left[V(\eta(t + \mathrm{d}t)) \middle| \eta(t)\right] - V(\eta(t))\right\}. \tag{10}$$

4 Main results

In this section, first the integral (5) is rewritten and then a sufficient condition for internal stability of (7) is provided, which are utilised to provide sufficient conditions for stochastic $\mathscr{H}_\infty$ of system (7). Further, a control synthesis via state feedback for the same is addressed.

4.1 Reformulation of the integral (5)

Proposition 1: The integral (5) can be rewritten as

$$\begin{aligned} J(T) = {} & x^\top(0) P_{r(\zeta_0, 0)} x(0) - \mathbb{E}\left[x^\top(T) P_{r(\zeta_T, T)} x(T)\right] \\ & + \mathbb{E}\left[\int_0^T \begin{bmatrix} x^\top(t) & v^\top(t) \end{bmatrix} \Lambda_{r(\zeta_t, t)} \begin{bmatrix} x(t) \\ v(t) \end{bmatrix} \mathrm{d}t\right], \end{aligned} \tag{11}$$

where

$$\Lambda_{r(\zeta_t = \kappa, t) = i} = \begin{bmatrix} A_i^\top P_i + P_i A_i + \sum_{j=1}^{n_S} \mu_{ij}^\kappa P_j & q_{12}\tilde{A}_i^\top P_i \tilde{B}_i + C_i^\top D_i + P_i B_i \\ + q_{11}\tilde{A}_i^\top P_i \tilde{A}_i + C_i^\top C_i & \\ \star & q_{22}\tilde{B}_i^\top P_i \tilde{B}_i - \gamma^2 \mathbb{I} + D_i^\top D_i \end{bmatrix}. \tag{12}$$

Proof: Consider a Lyapunov function $V(x(t), r(\zeta_t, t), \zeta_t) \triangleq x^\top(t) P_{r(\zeta_t, t)} x(t)$, where $P_{r(\zeta_t, t)} \succ 0$. By (10), for $i \in S$ and $\kappa \in \mathcal{K}$, $\mathscr{A}V(\eta(t))$ leads to

$$\begin{aligned} &\mathscr{A}V(x(t), r(\zeta_t = \kappa, t) = i, \zeta_t = \kappa) \\ &= \lim_{dt \to 0} \frac{1}{\mathrm{d}t}\Big\{\mathbb{E}\left[V(x(t + \mathrm{d}t), r(\zeta_{t+\mathrm{d}t}, t + \mathrm{d}t), \zeta_{t+\mathrm{d}t})\middle|(x(t),\right. \\ &\left. r(\zeta_t = \kappa, t) = i, \zeta_t = \kappa)\right] - V(x(t), r(\zeta_t = \kappa, t) = i, \zeta_t = \kappa)\Big\} \\ &= \lim_{\mathrm{d}t \to 0} \frac{1}{\mathrm{d}t}\Big\{\Big[\sum_{j=1}^{n_S} \Pr\{r(\zeta_{t+\mathrm{d}t}, t + \mathrm{d}t) = j \,|\, r(\zeta_t = \kappa, t) = i, \zeta_t = \kappa\} \\ &\mathbb{E}\left[x^\top(t + \mathrm{d}t) P_j x(t + \mathrm{d}t)\right]\Big] - \mathbb{E}\left[x^\top(t) P_i x(t)\right]\Big\} \\ &= \lim_{dt \to 0} \frac{1}{\mathrm{d}t}\Big\{\Big[\sum_{j=1, j \neq i}^{n_S} \mu_{ij}^\kappa\,\mathrm{d}t\,\mathbb{E}\left[x^\top(t + \mathrm{d}t) P_j x(t + \mathrm{d}t)\right]\Big] \\ &\quad + \left[(1 + \mu_{ii}^\kappa\,\mathrm{d}t)\mathbb{E}\left[x^\top(t + \mathrm{d}t) P_i x(t + \mathrm{d}t)\right]\right] - x^\top(t) P_i x(t)\Big\}, \end{aligned} \tag{13}$$

where $x(t + \mathrm{d}t) \simeq x(t) + A_i x(t)\,\mathrm{d}t + \tilde{A}_i x(t)\,\mathrm{d}w_1(t) + \tilde{B}_i v(t)\,\mathrm{d}w_2(t)$ $+ B_i v(t)\,\mathrm{d}t$. Using Itô's lemma [27], (13) is simplified to

$$\begin{aligned} &\mathscr{A}V(x(t), r(\zeta_t = \kappa, t) = i, \zeta_t = \kappa) \\ &= x^\top(t)\{A_i^\top P_i + P_i A_i + q_{11}\tilde{A}_i^\top P_i \tilde{A}_i + \sum_{j=1}^{n_S} \mu_{ij}^\kappa P_j\} x(t) \\ &\quad + \mathbb{E}[x^\top(t) P_i B_i v(t) + q_{12} x^\top(t) \tilde{A}_i^\top P_i \tilde{B}_i v(t) + q_{12} v^\top(t) \tilde{B}_i^\top P_i \tilde{A}_i x(t) \\ &\quad + q_{22} v^\top(t) \tilde{B}_i^\top P_i \tilde{B}_i v(t) + v^\top(t) B_i^\top P_i x(t)] \\ &= x^\top(t)\{A_i^\top P_i + P_i A_i + q_{11}\tilde{A}_i^\top P_i \tilde{A}_i + \sum_{j=1}^{n_S} \mu_{ij}^\kappa P_j\} x(t) \\ &\quad + \begin{bmatrix} x^\top(t) & v^\top(t) \end{bmatrix} \begin{bmatrix} q_{11}\tilde{A}_i^\top P_i \tilde{A}_i & q_{12}\tilde{A}_i^\top P_i \tilde{B}_i + P_i B_i \\ \star & q_{22}\tilde{B}_i^\top P_i \tilde{B}_i \end{bmatrix} \begin{bmatrix} x(t) \\ v(t) \end{bmatrix}. \end{aligned}$$

From (9), for any $i_0 \in \mathcal{K}$, we can write

$$\begin{aligned} &\mathbb{E}\left[V(x(t), r(\zeta_t, t), \zeta_t)|(x(0), r(\zeta_0 = i_0, 0), \zeta_0 = i_0)\right] \\ &\quad - V(x(0), r(\zeta_0 = i_0, 0), \zeta_0 = i_0) \\ &\quad = \sum_{k=0}^{k^*} \mathbb{E}\left[\int_{t \wedge \tau_{k-1}}^{t \wedge \tau_k} \mathscr{A}V(x(s), r(\zeta_s, s), \zeta_s)\,\mathrm{d}s \middle| (x(s), r(\zeta_s, s), \zeta_s)\right] \\ &\quad = \mathbb{E}\left[\int_0^t \mathscr{A}V(x(t), r(\zeta_t, t), \zeta_t)\,\mathrm{d}t \middle| (x(t), r(\zeta_t, t), \zeta_t)\right], \end{aligned} \tag{14}$$

where k^*, τ_{-1} and τ_{k^*} are given in (9) and noting that $\sum_{k=0}^{k^*} \int_{t \wedge \tau_{k-1}}^{t \wedge \tau_k} = \int_0^t$. Taking the expectation of (14) and letting the final time as T, we obtain

$$\begin{aligned} &\mathbb{E}\left[x^\top(T) P_{r(\zeta_T, T)} x(T)\right] - x^\top(0) P_{r(\zeta_0, 0)} x(0) \\ &\quad = \mathbb{E}\left[\int_0^T \mathscr{A}V(x(t), r(\zeta_t, t), \zeta_t)\,\mathrm{d}t\right] \\ &\quad = \mathbb{E}\left[\int_0^T x^\top(t)\{A_i^\top P_i + P_i A_i + q_{11}\tilde{A}_i^\top P_i \tilde{A}_i + \sum_{j=1}^{n_S} \mu_{ij}^\kappa P_j\} x(t)\,\mathrm{d}t\right] \\ &\quad + \mathbb{E}\left[\int_0^T \begin{bmatrix} x^\top(t) & v^\top(t) \end{bmatrix} \begin{bmatrix} q_{11}\tilde{A}_i^\top P_i \tilde{A}_i & q_{12}\tilde{A}_i^\top P_i \tilde{B}_i \\ & + P_i B_i \\ \star & q_{22}\tilde{B}_i^\top P_i \tilde{B}_i \end{bmatrix} \begin{bmatrix} x(t) \\ v(t) \end{bmatrix} \mathrm{d}t\right] \end{aligned}$$

for any $\zeta_t = \kappa$ and $r(\zeta_t = \kappa, t) = i$. Let $\mathscr{M}_{i\kappa} \triangleq A_i^\top P_i + P_i A_i + q_{11}\tilde{A}_i^\top P_i \tilde{A}_i + \sum_{j=1}^{n_S} \mu_{ij}^\kappa P_j$, $\mathscr{N}_i \triangleq \begin{bmatrix} q_{11}\tilde{A}_i^\top P_i \tilde{A}_i & q_{12}\tilde{A}_i^\top P_i \tilde{B}_i \\ & + P_i B_i \\ \star & q_{22}\tilde{B}_i^\top P_i \tilde{B}_i \end{bmatrix}$, and consider

$$J(T)+\mathbb{E}\left[x^{\top}(T)P_{r(\zeta_T,T)}x(T)\right]-x^{\top}(0)P_{r(\zeta_0,0)}x(0)$$
$$=\mathbb{E}\int_0^T\left\{\|z(t)\|^2-\gamma^2\|v(t)\|^2+x^{\top}(t)\mathscr{M}_{i\kappa}x(t)+\begin{bmatrix}x^{\top}(t) & v^{\top}(t)\end{bmatrix}\mathscr{N}_i\begin{bmatrix}x(t)\\ v(t)\end{bmatrix}\right\}\mathrm{d}t$$
$$=\mathbb{E}\int_0^T\left\{\|C_ix(t)+D_iv(t)\|^2-\gamma^2\|v(t)\|^2+x^{\top}(t)\mathscr{M}_{i\kappa}x(t)+\begin{bmatrix}x^{\top}(t) & v^{\top}(t)\end{bmatrix}\mathscr{N}_i\begin{bmatrix}x(t)\\ v(t)\end{bmatrix}\right\}\mathrm{d}t,$$

which, after rearrangement, will lead to (11). □

4.2 Internal stability

Proposition 2: If $\exists\, P_i \succ 0, \forall \kappa \in \mathscr{K}$ and $\forall i \in S$, such that

$$A_i^{\top}P_i+P_iA_i+q_{11}\tilde{A}_i^{\top}P_i\tilde{A}_i+\sum_{j=1}^{n_S}\mu_{ij}^{\kappa}P_j \prec 0, \tag{15}$$

then system (7) is internally stable.

Proof: Consider $V(x(t), r(\zeta_t,t),\zeta_t) \triangleq x^{\top}(t)P_{r(\zeta_t,t)}x(t)$. From (13), we obtain $\mathscr{A}V$ as

$$\begin{aligned}&\mathscr{A}V(x(t), r(\zeta_t=\kappa,t)=i,\zeta_t=\kappa)\\ &=\lim_{\mathrm{d}t\to 0}\frac{1}{\mathrm{d}t}\left\{\left[\sum_{j=1,j\neq i}^{n_S}\mu_{ij}^{\kappa}\,\mathrm{d}t\,\mathbb{E}\left[x^{\top}(t+\mathrm{d}t)P_jx(t+\mathrm{d}t)\right]\right]\right.\\ &\left.+\left[(1+\mu_{ii}^{\kappa}\mathrm{d}t)\mathbb{E}\left[x^{\top}(t+\mathrm{d}t)P_ix(t+\mathrm{d}t)\right]\right]-x^{\top}(t)P_ix(t)\right\},\end{aligned} \tag{16}$$

where $x(t+\mathrm{d}t)\simeq x(t)+A_ix(t)\,\mathrm{d}t+\tilde{A}_ix(t)\,\mathrm{d}w_1(t)$ since $v(t)=0$. Using Itô's lemma, (16) is simplified as

$$\begin{aligned}&\mathscr{A}V(x(t), r(\zeta_t=\kappa,t)=i,\zeta_t=\kappa)\\ &=x^{\top}(t)\left\{A_i^{\top}P_i+P_iA_i+q_{11}\tilde{A}_i^{\top}P_i\tilde{A}_i+\sum_{j=1}^{n_S}\mu_{ij}^{\kappa}P_j\right\}x(t).\end{aligned} \tag{17}$$

From (15), define

$$A_i^{\top}P_i+P_iA_i+q_{11}\tilde{A}_i^{\top}P_i\tilde{A}_i+\sum_{j=1}^{n_S}\mu_{ij}^{\kappa}P_j \triangleq -W_{\kappa i} \tag{18}$$

with $-W_{\kappa i} \prec 0$. Thus, from (15), $\mathscr{A}V$ (17) simplifies to

$$\begin{aligned}&\mathscr{A}V(x(t), r(\zeta_t=\kappa,t)=i,\zeta_t=\kappa)\\ &\le -x^{\top}(t)W_{\kappa i}x(t)\\ &\le -\min_{\kappa\in\mathscr{K},i\in S}\{\lambda_{\min}(W_{\kappa i})\}x^{\top}(t)x(t).\end{aligned} \tag{19}$$

From (9), for any $i_0 \in \mathscr{K}$, we can write

$$\begin{aligned}&\mathbb{E}\left[V(x(t), r(\zeta_t,t),\zeta_t)\,|\,(x(0), r(\zeta_0=i_0,0),\zeta_0=i_0)\right]\\ &-V(x(0), r(\zeta_0=i_0,0),\zeta_0=i_0)\\ &=\sum_{k=0}^{k^*}\mathbb{E}\left[\int_{t\wedge\tau_{k-1}}^{t\wedge\tau_k}\mathscr{A}V(x(s), r(\zeta_s,s),\zeta_s)\,\mathrm{d}s\,\middle|\,(x(s), r(\zeta_s,s),\zeta_s)\right].\end{aligned} \tag{20}$$

where k^*, τ_{-1} and τ_{k^*} are given in (9). In view of remark 3, let $\{i_0, i_1, i_2, \dots\} \in \mathscr{K}$ be the sequential state values of ζ_t. Then (20) can be recast as (see (21)) . By (19), the above term reduces to

$$\begin{aligned}&\mathbb{E}\left[V(x(t), r(\zeta_t,t),\zeta_t)\,|\,(x(0), r(\zeta_0=i_0,0),\zeta_0=i_0)\right]\\ &-V(x(0), r(\zeta_0=i_0,0),\zeta_0=i_0)\\ &\le -\min_{\kappa\in\mathscr{K},i\in S}\{\lambda_{\min}(W_{\kappa i})\}\left(\mathbb{E}\left[\int_0^{\tau_0}\|x(s)\|^2\,\mathrm{d}s\right]\right.\\ &\left.+\mathbb{E}\left[\int_{\tau_0}^{\tau_1}\|x(s)\|^2\,\mathrm{d}s\right]+\cdots+\mathbb{E}\left[\int_{t\wedge\tau_{k^*-1}}^{t\wedge\tau_{k^*}}\|x(s)\|^2\,\mathrm{d}s\right]\right)\end{aligned} \tag{22}$$

By noting $\sum_{k=0}^{k^*}\int_{t\wedge\tau_{k-1}}^{t\wedge\tau_k}=\int_0^t$, (22) is simplified as

$$\begin{aligned}&\mathbb{E}\left[V(x(t), r(\zeta_t,t),\zeta_t)\,|\,(x(0), r(\zeta_0=i_0,0),\zeta_0=i_0)\right]\\ &-V(x(0), r(\zeta_0=i_0,0),\zeta_0=i_0)\\ &\le -\min_{\kappa\in\mathscr{K},i\in S}\{\lambda_{\min}(W_{\kappa i})\}\mathbb{E}\left[\int_0^t\|x(s)\|^2\,\mathrm{d}s\right].\end{aligned}$$

Thus we obtain,

$$\begin{aligned}&\min_{\kappa\in\mathscr{K},i\in S}\{\lambda_{\min}(W_{\kappa i})\}\mathbb{E}\left[\int_0^t\|x(s)\|^2\,\mathrm{d}s\right]\\ &\le V(x(0), r(\zeta_0=i_0,0),\zeta_0=i_0)\\ &-\mathbb{E}\left[V(x(t), r(\zeta_t,t),\zeta_t)\,|\,(x(0), r(\zeta_0=i_0,0),\zeta_0=i_0)\right]\\ &\le V(x(0), r(\zeta_0=i_0,0),\zeta_0=i_0),\end{aligned}$$

which is recast as

$$\mathbb{E}\left[\int_0^t\|x(s)\|^2\,\mathrm{d}s\right]\le\frac{V(x(0), r(\zeta_0=i_0,0),\zeta_0=i_0)}{\min_{\kappa\in\mathscr{K},i\in S}\{\lambda_{\min}(W_{\kappa i})\}}.$$

For $t\to\infty$,

$$\mathbb{E}\left[\int_0^{\infty}\|x(s)\|^2\,\mathrm{d}s\right]\le\frac{V(x(0), r(\zeta_0=i_0,0),\zeta_0=i_0)}{\min_{\kappa\in\mathscr{K},i\in S}\{\lambda_{\min}(W_{\kappa i})\}}<\infty.$$

Hence system (7) is internally stable. □

4.3 Stochastic $\mathscr{H}_\infty$

In this section, a proposition for verifying the stochastic $\mathscr{H}_\infty$ condition (4) is provided.

Proposition 3: If $\exists\, P_i \succ 0, \forall\kappa\in\mathscr{K}$ and $\forall i\in S$, satisfying

$$\Lambda_{r(\zeta_t=\kappa,t)=i}\prec 0, \tag{23}$$

$$\begin{aligned}&\mathbb{E}\left[V(x(t), r(\zeta_t,t),\zeta_t)\,|\,(x(0), r(\zeta_0=i_0,0),\zeta_0=i_0)\right]\\ &-V(x(0), r(\zeta_0=i_0,0),\zeta_0=i_0)\\ &=\mathbb{E}\left[\int_0^{\tau_0}\mathscr{A}V(x(s), r(\zeta_s=i_0,s),\zeta_s=i_0)\,\mathrm{d}s\,\middle|\,(x(s), r(\zeta_s=i_0,s),\zeta_s=i_0)\right]\\ &+\mathbb{E}\left[\int_{\tau_0}^{\tau_1}\mathscr{A}V(x(s), r(\zeta_s=i_1,s),\zeta_s=i_1)\,\mathrm{d}s\,\middle|\,(x(s), r(\zeta_s=i_1,s),\zeta_s=i_1)\right]\\ &+\cdots+\mathbb{E}\left[\int_{t\wedge\tau_{k^*-1}}^{t\wedge\tau_{k^*}}\mathscr{A}V(x(s), r(\zeta_s,s),\zeta_s)\,\mathrm{d}s\,\middle|\,(x(s), r(\zeta_s,s),\zeta_s)\right].\end{aligned} \tag{21}$$

then system (7) is internally stable while satisfying stochastic $\mathscr{H}_\infty$ condition (4).

Proof: Observe that, from (23) and (12), we can write

$$A_i^{\top} P_i + P_i A_i + \sum_{j=1}^{n_S} \mu_{ij}^{\kappa} P_j + q_{11} \tilde{A}_i^{\top} P_i \tilde{A}_i + C_i^{\top} C_i \prec 0, \tag{24}$$

which implies

$$A_i^{\top} P_i + P_i A_i + \sum_{j=1}^{n_S} \mu_{ij}^{\kappa} P_j + q_{11} \tilde{A}_i^{\top} P_i \tilde{A}_i \prec 0, \tag{25}$$

thus from (15), system (7) is internally stable. Now from (11), (12), and (23), the integral $J(T)$ (5) simplifies to

$$\begin{aligned}&\int_0^T \mathbb{E}[\ \| z(t) \|^2 - \gamma^2 \| v(t) \|^2\ ]\\ &\le x^{\top}(0) P_{r(\zeta_0, 0)} x(0) - \mathbb{E}\big[x^{\top}(T) P_{r(\zeta_T, T)} x(T)\big]\\ &\le 0,\end{aligned}$$

which is due to zero initial condition for verifying (4). □

4.4 State-feedback control

In this section, control synthesis while satisfying stochastic $\mathscr{H}_\infty$ is addressed.

With the non-zero control input, we get system (7) dynamics as

$$\begin{aligned}\mathrm{d}x(t) &= A_{r(\zeta_t, t)} x(t)\,\mathrm{d}t + \tilde{A}_{r(\zeta_t, t)} x(t)\,\mathrm{d}w_1(t) + \tilde{B}_{r(\zeta_t, t)} v(t)\,\mathrm{d}w_2(t)\\ &\quad + B_{r(\zeta_t, t)} v(t)\,\mathrm{d}t + E_{r(\zeta_t, t)} u(t)\,\mathrm{d}t\end{aligned} \tag{26a}$$

$$z(t) = C_{r(\zeta_t, t)} x(t) + D_{r(\zeta_t, t)} v(t) + F_{r(\zeta_t, t)} u(t)\,\mathrm{d}t\,. \tag{26b}$$

Assuming the availability of the mode process $r(\zeta_t, t)$, consider state-feedback control law

$$u(t) = K_{r(\zeta_t, t)} x(t), \tag{27}$$

where $K_{r(\zeta_t, t)} \in \mathbb{R}^{n_u \times n_x}$ for $r(\zeta_t, t) \in S$. Hence the controlled dynamics lead to

$$\begin{aligned}\mathrm{d}x(t) &= \bar{A}_{r(\zeta_t, t)} x(t)\,\mathrm{d}t + \tilde{A}_{r(\zeta_t, t)} x(t)\,\mathrm{d}w_1(t) + \tilde{B}_{r(\zeta_t, t)} v(t)\,\mathrm{d}w_2(t)\\ &\quad + B_{r(\zeta_t, t)} v(t)\,\mathrm{d}t + E_{r(\zeta_t, t)} u(t)\,\mathrm{d}t,\end{aligned} \tag{28a}$$

$$z(t) = \bar{C}_{r(\zeta_t, t)} x(t) + D_{r(\zeta_t, t)} v(t) + F_{r(\zeta_t, t)} u(t)\,\mathrm{d}t, \tag{28b}$$

where

$$\bar{A}_{r(\zeta_t, t)} = A_{r(\zeta_t, t)} + E_{r(\zeta_t, t)} K_{r(\zeta_t, t)} \tag{29}$$

and

$$\bar{C}_{r(\zeta_t, t)} = C_{r(\zeta_t, t)} + F_{r(\zeta_t, t)} K_{r(\zeta_t, t)}\,. \tag{30}$$

Proposition 4: If $\exists\ \Phi_i \succ 0$ and Υ_i, $\forall \kappa \in \mathscr{K}$ and $\forall i \in S$, such that (33) holds, where

$$\Omega_{\kappa i} = \left[\sqrt{\mu_{i1}^{\kappa}}\Phi_i \ldots \sqrt{\mu_{ii-1}^{\kappa}}\Phi_i,\ \sqrt{\mu_{ii+1}^{\kappa}}\Phi_i \ldots \sqrt{\mu_{in_S}^{\kappa}}\Phi_i\right] \tag{31}$$

$$\Gamma_i = \mathrm{diag}\{\Phi_1 \cdots \Phi_{i-1}, \Phi_{i+1} \cdots \Phi_{n_S}\}, \tag{32}$$

then system (28) achieves stochastic $\mathscr{H}_\infty$ condition (4) by control law (27), and the stabilising controller is given by $K_i = \Upsilon_i \Phi_i^{-1}$.

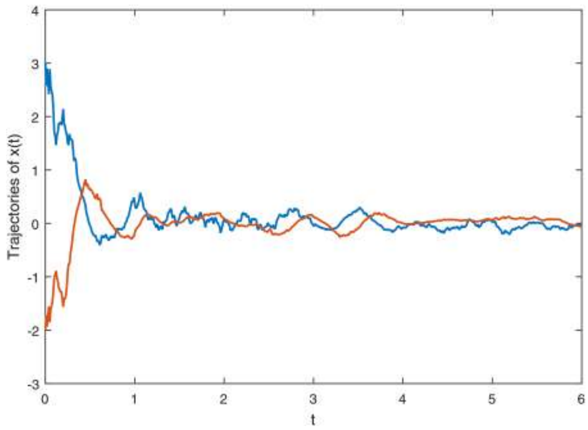

Fig. 1 *State trajectories*

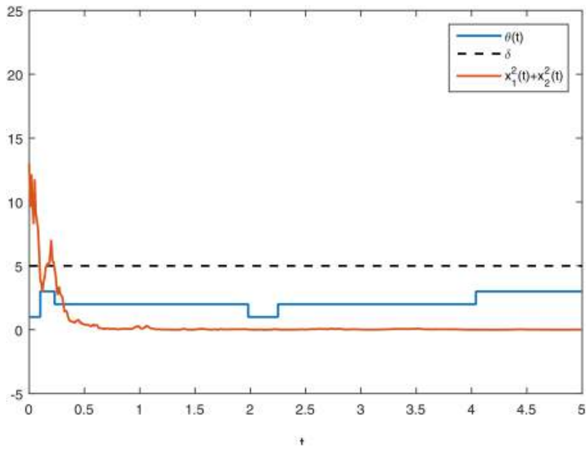

Fig. 2 *Mode process $\theta(t)$*

Proof: From proposition 3, controlled system (28) satisfies the stochastic $\mathscr{H}_\infty$ condition if (34) is satisfied $\forall \kappa \in \mathscr{K}$ and $\forall i \in S$, where $\bar{A}_i = A_i + E_i K_i$ and $\bar{C}_i = C_i + F_i K_i$. Now the Schur complement of (34) leads to (35), where $q_{ij}^{-1} = (Q^{-1})_{ij}$. Let $\Phi_i = P_i^{-1}$, $\Upsilon_i = K_i \Phi_i$ and applying the congruent transformation of (35) with $\mathrm{diag}\{P_i^{-1}, \mathbb{I}, \mathbb{I}, \mathbb{I}, \mathbb{I}\}$ leads to (36), where $\Omega_{\kappa i}$ is given in (31), Γ_i is given by (32) and controller gains are computed by $K_i = \Upsilon_i \Phi_i^{-1}$. □

5 Example

Let $x(t) \in \mathbb{R}^2$ and scalar $u(t)$ in (28) with $\theta(t) \in S := \{1, 2, 3\}$ and

$$A_1 = \begin{bmatrix} 1 & -1 \\ 2 & 0 \end{bmatrix},\quad A_2 = \begin{bmatrix} 2 & -4 \\ 8 & 0 \end{bmatrix},\quad A_3 = \begin{bmatrix} 3 & 4 \\ 0 & -1 \end{bmatrix},$$

$$\tilde{A}_1 = \tilde{A}_2 = \tilde{A}_3 = -\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},\quad B_1 = B_2 = B_3 = [0 \quad 1]^{\top},$$

$$\tilde{B}_1 = \tilde{B}_2 = \tilde{B}_3 = [1 \quad 0]^{\top},\quad C_1 = C_2 = C_3 = [0 \quad 1],$$

$$\begin{aligned}&D_1 = D_2 = D_3 = F_1 = F_2 = F_3 = 1,\quad E_1 = [4 \quad 0]^{\top},\\ &E_2 = [2 \quad 0]^{\top},\quad E_3 = [4 \quad 0]^{\top}.\end{aligned}$$

Consider $\theta(t)$ (2) with $\mathscr{C}_1 \triangleq \{x(t) \in \mathbb{R}^2 : x_1^2(t) + x_2^2(t) < \delta\}$, $\mathscr{C}_2 \triangleq \{x(t) \in \mathbb{R}^2 : x_1^2(t) + x_2^2(t) \ge \delta\}$, $\delta = 5$, and $\mathscr{K} := \{1, 2\}$. Consider

$$(\mu_{ij}^1)_{3\times 3} = \begin{bmatrix} -8 & 3 & 5 \\ 6 & -11 & 5 \\ 3 & 3 & -6 \end{bmatrix},\quad (\mu_{ij}^2)_{3\times 3} = \begin{bmatrix} -7 & 5 & 2 \\ 3 & -4 & 1 \\ 6 & 6 & -12 \end{bmatrix}.$$

$$\begin{bmatrix} A_i\Phi_i + \Phi_i A_i^\top + E_i\Upsilon_i + \Upsilon_i^\top E_i + \mu_{ii}\Phi_i & B_i & \Phi_i C_i^\top + \Upsilon_i F_i^\top & \Phi_i \tilde{A}_i^\top & 0 & \Omega_{\kappa i} \\ \star & -\gamma^2\mathbb{I} & D_i^\top & 0 & \tilde{B}_i^\top & 0 \\ \star & \star & -\mathbb{I} & 0 & 0 & 0 \\ \star & \star & \star & -q_{11}^{-1}\Phi_i & -q_{12}^{-1}\Phi_i & 0 \\ \star & \star & \star & \star & -q_{22}^{-1}\Phi_i & 0 \\ \star & \star & \star & \star & \star & -\Gamma_i \end{bmatrix} \prec 0 \quad (33)$$

$$\begin{bmatrix} \bar{A}_i^\top P_i + P_i\bar{A}_i + \sum_{j=1}^{n_S}\mu_{ij}^\kappa P_j + q_{11}\tilde{A}_i^\top P_i\tilde{A}_i + \bar{C}_i^\top\bar{C}_i & q_{12}\tilde{A}_i^\top P_i\tilde{B}_i + \bar{C}_i^\top D_i + P_i B_i \\ \star & q_{22}\tilde{B}_i^\top P_i\tilde{B}_i - \gamma^2\mathbb{I} + D_i^\top D_i \end{bmatrix} \prec 0 \quad (34)$$

$$\begin{bmatrix} (A_i + E_iK_i)^\top P_i + P_i(A_i + E_iK_i) + \sum_{j=1}^{n_S}\mu_{ij}^\kappa P_j & P_iB_i & (C_i + F_iK_i)^\top & \tilde{A}_i^\top & 0 \\ \star & -\gamma^2\mathbb{I} & D_i^\top & 0 & \tilde{B}_i^\top \\ \star & \star & -\mathbb{I} & 0 & 0 \\ \star & \star & \star & -q_{11}^{-1}P_i^{-1} & -q_{12}^{-1}P_i^{-1} \\ \star & \star & \star & \star & -q_{22}^{-1}P_i^{-1} \end{bmatrix} \prec 0 \quad (35)$$

$$\begin{bmatrix} A_i\Phi_i + \Phi_i A_i^\top + E_i\Upsilon_i + \Upsilon_i^\top E_i + \mu_{ii}\Phi_i & B_i & \Phi_i C_i^\top + \Upsilon_i F_i^\top & \Phi_i \tilde{A}_i^\top & 0 & \Omega_{\kappa i} \\ \star & -\gamma^2\mathbb{I} & D_i^\top & 0 & \tilde{B}_i^\top & 0 \\ \star & \star & -\mathbb{I} & 0 & 0 & 0 \\ \star & \star & \star & -q_{11}^{-1}\Phi_i & -q_{12}^{-1}\Phi_i & 0 \\ \star & \star & \star & \star & -q_{22}^{-1}\Phi_i & 0 \\ \star & \star & \star & \star & \star & -\Gamma_i \end{bmatrix} \prec 0 \quad (36)$$

Let $\gamma = 4$. From proposition 4, a solution of LMIs (33) can be given by

$$\Phi_1 = \begin{bmatrix} 0.9153 & -0.0989 \\ -0.0989 & 0.5253 \end{bmatrix}, \quad \Phi_2 = \begin{bmatrix} 0.5495 & -0.1189 \\ -0.1189 & 0.6267 \end{bmatrix}, \quad \Phi_3 = \begin{bmatrix} 0.7773 & -0.1068 \\ -0.1068 & 0.5942 \end{bmatrix},$$

$$\Upsilon_1 = [-1.2497 \quad -0.2468], \quad \Upsilon_2 = [-1.5220 \quad -0.6949], \quad \Upsilon_3 = [-1.1583 \quad -0.4971]$$

and the resulting controller gains are: $K_1 = [-1.4454 \quad -0.7402]$, $K_2 = [-3.1382 \quad -1.7040]$, $K_3 = [-1.6456 \quad -1.1322]$.

A sample evolution of the system states with initial mode as 2 and $x_1(0) = 3$, $x_2(0) = -2$ is shown in Fig. 1. Also, an evolution of $\theta(t)$ is depicted in Fig. 2. From the Monte Carlo simulations of 1000 runs, $\| \mathbb{L} \|$ is obtained as $3.0853 < \gamma$.

(see (33))
(see (34))
(see (35))
(see (36))

6 Conclusions

In this paper, a stochastic $\mathscr{H}_\infty$ analysis for a SJLS affected by state-dependent noise and stochastic perturbations is addressed. The underlying state-dependent jump process is assumed to have dissimilar transition rates among sets of state evolution space. For no input case, using Dynkin's formula and Itô's lemma, tractable conditions for stochastic $\mathscr{H}_\infty$ are obtained by employing Lyapunov's second method. Further, the results are stretched to state-feedback control synthesis. As a perspective, the obtained results can be applied to fault tolerant control systems affected by state-dependent failures.

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